



Steered adaptive subarray beamforming

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Defence R&D Canada

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ABSTRACT

The performance of the steered, minimum-variance, subarray beamformer is examined in some detail. It is demonstrated that improved performance is possible through the use of a randomised subarray geometry. It is also demonstrated that the beamformer is susceptible to multipath problems in both the narrowband and wideband modes. A series of simulation experiments allows its performance to be evaluated against that of other adaptive and non-adaptive beamformers.

RÉSUMÉ

Les performances du formeur de faisceaux de sous-réseau dirigé à variance minimale sont soumises à un examen assez approfondi. Il est démontré que des performances améliorées sont possibles grâce à l'utilisation d'une géométrie de sous-réseau randomisée. Il est également démontré que le formeur de faisceaux est sujet à des problèmes de propagation par trajets multiples, tant en bande étroite qu'en large bande. Une série d'expériences de simulation permet d'évaluer ses performances par comparaison avec celles d'autres formeurs de faisceaux adaptatifs et non adaptatifs.

Executive summary

The steered-minimum-variance subarray (STMV-SA) beamformer is a partially adaptive beamformer whose number of adaptive degrees of freedom may be much less than the number of sensors in the array. Given that the performance of such a beamformer improves as the ratio of the number of adaptive degrees of freedom to the time-bandwidth product of the processed data decreases, such a beamformer is of interest in situations where the observation time is short.

The three key facets of the STMV-SA beamformer are: (1) a reduction in the number of adaptive degrees of freedom by utilizing in the adaptive process only the outputs of a relative small number of "delay-and-sum" beamformers, each formed on taking a different subset of the sensors of the original array, here assumed to be a uniform line-array; (2) the concomitant reduction in dimensionality and computational loading of the adaptive processing; and (3) the utilization of wideband input data through the use of the steered-covariance matrix formulation, which in essence implies a simple pre-steering of the array to the bearing of interest.

It was found that the geometry of the sub arrays could have a significant effect, particularly if the original array had failed or had missing sensors. Overall the best arrangement was deemed to be one in which the delay-and-sum subarrays were formed from a randomized selection of the original sensors.

It was demonstrated that the STMV-SA beamformer was a viable beamformer in both the narrowband (NB) mode, where the signals of interest might be tonals, and in the wideband mode where the signals of interest might be, say, FM chirps.

Notwithstanding the above, it was found that the STMV-SA beamformer is susceptible to the usual problems which bedevil minimum-variance beamformers, such as (unknown) sensor perturbations and, more particularly, the presence of multipath conditions where multiple signals are co-located both in time and frequency. The multipath problem was shown to exist in both the narrowband and wideband applications.

In terms of finite-data performance, it was shown that some previously published results relating to the finite-data, statistical performance of minimum-variance beamformer, including output SNR, were applicable to the STMV-SA beamformer with only simple modification. It was also demonstrated that the NB mode of operation offered surprisingly good results, better, in fact, than the corresponding narrowband benchmark with the same time-bandwidth product and number of degrees of freedom.

The Dominant Mode Rejection Beamformer of Owsley was found not to be compatible with the use of the underlying wideband steered covariance matrix of the STMV approach.

A software package was written to simulate the "real-world" performance of various beamformers, including the STMV-SA. In the experiments described herein, it would seem that if there are no missing sensors, then the fixed weight, Hamming-shaded beamformer is a

reasonable choice: its performance is competitive, with no finite-data problems, while its implementation is straightforward. In the case of missing sensors, the best overall performer, though not the most computationally efficient, was found to be the ALPINEX beamformer introduced at DREA some years ago. Neither the Hamming nor the ALPINEX beamformer is sensitive to multipath conditions. Unfortunately, none of the experiments revealed the STMV-SA processor to be clearly superior in performance.

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Sommaire

Le conformateur de faisceau de sous-réseau orienté à variance minimale (STMV-SA) est un conformateur de faisceau partiellement adaptatif dont le nombre de degrés de liberté d'adaptation peut être nettement inférieur au nombre de capteurs du réseau. Étant donné que les performances d'un tel conformateur de faisceau s'améliorent à mesure que le rapport du nombre de degrés de liberté d'adaptation au produit temps-largeur de bande des données traitées diminue, un tel conformateur de faisceau est d'intérêt dans des situations où le temps d'observation est bref.

Les trois aspects clés du conformateur de faisceau STMV-SA sont : (1) réduction du nombre de degrés de liberté d'adaptation en n'utilisant dans le processus d'adaptation que les signaux de sortie d'un nombre relativement petit de conformateurs de faisceau à « retard et sommation », alimentés chacun par un sous-ensemble différent de capteurs du réseau d'origine, supposé ici être un réseau linéaire uniforme; (2) réduction concomitante de la dimensionnalité et de la charge de calcul du traitement d'adaptation; (3) utilisation de données d'entrée à large bande grâce à l'utilisation de la formulation de la matrice de covariance d'orientation, ce qui implique essentiellement la simple pré-orientation du réseau vers le relèvement d'intérêt.

Il a été déterminé que la géométrie des sous-réseaux pourrait avoir un effet considérable, notamment lorsque le réseau d'origine était tombé en panne ou avait des capteurs manquants. Tout compte fait, la configuration dans laquelle les sous-réseaux de retard et sommation étaient constitués d'une sélection randomisée des capteurs d'origine était considérée comme la configuration optimale.

Il a été démontré que le conformateur de faisceau STMV-SA était un conformateur de faisceau viable tant en mode bande étroite (BE), où les signaux d'intérêt peuvent être des tonalités, qu'en mode large bande, où les signaux d'intérêt peuvent être, par exemple, des impulsions comprimées en FM.

Malgré ce qui précède, nous avons constaté que le conformateur de faisceau STMV-SA est sujet aux problèmes usuels qui gênent les conformateurs de faisceau à variance minimale, tels que les perturbations des capteurs (d'origine inconnue) et, plus particulièrement, la présence de conditions d'interférence entre trajets multiples dans lesquelles des signaux multiples se produisent en même temps et à la même fréquence. Il a été démontré que le problème des trajets multiples existait tant dans les applications à bande étroite que dans les applications à large bande.

Pour ce qui est des performances avec des données finies, il a été démontré que certains résultats antérieurs en rapport avec les performances statistiques avec des données finies du conformateur de faisceau à variance minimale, y compris le rapport S/B de sortie, étaient applicables au conformateur de faisceau STMV-SA sous réserve d'une simple modification seulement. Il a également été démontré que le mode de fonctionnement BE donnait des résultats d'une qualité insoupçonnée; en fait, les résultats étaient supérieurs à la référence de

bande étroite correspondante avec le même produit temps-largeur de bande et le même nombre de degrés de liberté.

Il a été déterminé que le conformateur de faisceau à réjection du mode dominant d'Owsley n'était pas compatible avec l'utilisation de la matrice de covariance d'orientation à large bande sous-jacente du système STMV.

Nous avons préparé un ensemble de logiciels pour simuler les performances en conditions réelles de divers conformateurs de faisceau, y compris le STMV-SA. D'après les expériences décrites ici, il semblerait que, s'il n'y a pas de capteurs manquants, le conformateur de faisceau de poids fixe, masqué par réseau de Hamming soit un choix raisonnable : ses performances sont compétitives, sans problèmes au niveau des données finies, tandis que sa mise en œuvre est peu complexe. Dans le cas de capteurs manquants, le modèle offrant les meilleures performances globales, mais non les calculs les plus efficaces, s'est avéré le conformateur de faisceau ALPINEX introduit au CRDA il y a quelques années. Ni le conformateur de faisceau de Hamming ni le conformateur de faisceau ALPINEX ne sont sensibles aux conditions de trajets multiples. Malheureusement, aucune des expériences n'a révélé des performances nettement supérieures du processeur du STMV-SA.

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STEERED ADAPTIVE SUBARRAY BEAMFORMING

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1. INTRODUCTION

This report is the result of an investigation into some of the attributes of the adaptive, steered-minimum-variance, subarray beamformer (STMV-SA) recently introduced at DREA by Stergiopoulos. Much of the work was performed in conjunction with the Department of Electrical Engineering at DalTech (formerly TUNS) e.g. [2-2, 2-3, 2-4]. The method is one of the class of partially-adaptive beamformers, meaning that the number of adaptive degrees of freedom may be considerably fewer than the number of sensors in the array, here assumed to be a uniform line-array. Given that the performance of such beamformers is inextricably linked to the ratio of the number of adaptive degrees of freedom to the time-bandwidth product of the processed data, then such beamformers are attractive when used with short observation times. In the application at hand, the beamformer's weights are computed from a single, relatively short, wideband (WB) observation, or snapshot. Beamformers of this type with "no memory" of previous snapshots are said to possess the property of "instantaneous convergence" [2-3].

It is very important at the outset to grasp the concept that the STMV-SA beamformer is based on a covariance matrix estimate obtained from wideband (WB) data. However, the target application may be narrowband (NB), for instance where the data contains tonal signals, or WB, where the data may contain WB signals of interest, FM chirps, say. The adaptive process is based on the familiar minimum-variance distortionless-response (MVDR) beamformer but the dimensionality of the problem is much reduced by using, as inputs to the adaptive process, the outputs of a relatively small number of "delay-and-sum" beamformers, each of which is formed from a different subset of contiguous sensors in the original array.

This concept is further linked to the notion of forming a WB steered-covariance matrix which simply involves pre-steering the array to a given direction of interest; all signals emanating from this chosen direction have a spatial frequency of zero measured across the array, akin to a signal arriving at an unsteered line-array from broadside, and it is precisely this condition which allows WB signals to be "coherently" processed by the ostensibly narrowband MVDR process.

So we have in the STMV-SA beamformer a technique which has the desirable properties of operating with short observation times, yet still performing in a statistically

stable manner, while simultaneously offering a reduced computational load due to the limited dimensionality of the adaptive process. The task at hand is to investigate the performance of the STMV-SA beamformer in some depth.

We begin in Chapter 2 with an investigation of whether or not the delay-and-sum subarrays should comprise overlapped subsets of sensors from the original array. We also meet for the first time the issue of multipath arrivals which implies the presence of signals which are co-located in both time and frequency. The multipath problem has long been known to be troublesome for MVDR-based beamformers.

In Chapter 3 we deal with some common problems in real-world beamforming including sensor perturbation effects, missing sensors, and the impact of finite coherence lengths for our arrivals. In the case of missing sensors, we note potential problems associated with each subarray's being formed from an equal number of contiguous sensors in the original array.

Chapters 2 and 3 are both based on the simplifying assumption of infinite data (equivalently, infinite-time) where the underlying steered covariance matrix can be written out explicitly and the ensuing simulations and/or analysis are straightforward. However, such an approach can hide as much as it reveals. In Chapter 4 we begin to look at the impact of finite data on the STMV-SA beamformer and the concomitant decrease in performance, generally referred to as the "misadjustment phenomenon" associated with errors in estimating the adaptive weights from what must perforce be an estimate of the underlying covariance matrix. We recall that a key rationale behind the STMV-SA beamformer lies in its ability to operate successfully with limited amounts of data.

Another recently proposed beamformer with the attribute of being useful in conjunction with short observation times is the so-called "Dominant-Mode Rejection Beamformer" of Owsley [5-1, 5-2]. Originally designed for use in NB processing, we explore in Chapter 5 its utility in connection with our WB STCM matrix. The results are very disappointing and reveal that the two methods are basically incompatible.

Finally, in Chapter 6 we present the results of some simulation experiments designed to compare the performance of several beamformers, adaptive and non-adaptive, in a fairly wide-ranging set of "real-world" scenarios. The results are interesting. The report ends with some conclusions in Chapter 7.

2. INFINITE-DATA PERFORMANCE OF THE STMV-SA BEAMFORMER: OVERLAPPED OR CONTIGUOUS SUBARRAYS?

2.1 Introduction

The current version of STMV-SA beamforming as implemented at DREA uses heavily overlapped subarrays (SA's), with an inter-subarray offset of just one sensor. On the other hand, Owsley's 1987 report [2-1], ostensibly the basis of DREA's efforts, uses non-overlapped, but contiguous subarrays where the offset equals the number of sensors in each subarray. It is obviously of interest to examine the relative performance of these two approaches and this is the quintessential task at hand in this chapter.

We start with an overview of the STMV-SA process and then set up a framework under which the two methods may usefully be compared. The problem is made more complicated as parameters like the number, strength and location of interferers must be considered, as must the number of subarrays. Additionally, one must carefully distinguish between cases where the signal and interferences(s) may be co-located in frequency and others where there is no frequency overlap. Similarly, notwithstanding the fact that the STMV technique is essentially wideband, there is the issue of whether the target application is primarily narrowband (NB) or wideband (WB). Other issues, such as the effect of multi-path conditions, will be discussed as they arise.

2.2 STMV Beamforming

We assume we have a uniform line-array comprising N omni-sensors. A T -sec observation (time-domain snapshot) is recorded with a sampling rate of F_s Hz simultaneously at each sensor and Fast Fourier transformed. A frequency-domain snapshot vector, $\mathbf{x}_k$, dimension $N \times 1$, is created at frequency f_k on collecting the N complex Fourier coefficients at this frequency. [Note that this assumes that the frequency f_k falls at one of the FFT bin frequencies, $f_k = k T^{-1} \leq F_s/2$, k integer].

At the heart of STMV processing is the notion that the array is pre-steered to the direction of interest. If this is at a bearing of θ_0 with respect to broadside then it is equivalent, recalling that a time-domain delay is equivalent to a linear phase slope in the temporal frequency domain, to multiplying our snapshot $\mathbf{x}_k$ by a diagonal matrix, $\mathbf{D}(\theta_0, f_k)$, whose nn^{th} entry is

$$D_{nn}(\theta_0, f_k) = \exp(-j2\pi f_k n d \sin(\theta_0) / c), \quad n = 0, \dots, N-1$$

where d is the inter-element spacing and c is the speed of propagation. Note that in the above expression we can define a spatial frequency $\nu_k = 2\pi f_k d \sin(\theta_0) / c$ [rad/m] which pertains to a signal at frequency f_k emanating from the signal direction. Hence pre-steering is equivalent to spatial-frequency shifting. After pre-steering, the spatial frequency of a signal from the steering direction is always zero, at all temporal frequencies. This is the key to understanding the STMV method. In a non-steered array, it is equivalent to choosing broadside as the direction of interest.

It is convenient to concentrate on the processing steps *following* the pre-steering operation as this greatly simplifies the notation. The pre-steered snapshot vector is written as $\mathbf{y}_k = \mathbf{D}(\theta_0, f_k) \mathbf{x}_k$ and we will henceforth focus our attention on the pre-steered vector $\mathbf{y}_k$.

A salient feature of the STMV method is its ability to form a vector of sensor weights from a single time-series snapshot and hence it is sometimes referred to as having "quasi-instantaneous convergence" [2-2, 2-3]. This is a direct consequence of the fact that multiple temporal frequency snapshot vectors (NB) are available from a single (WB) time-series observation, and each of these, after pre-steering, has a spatial frequency component of zero in the chosen steering direction.

Suppose we have a contiguous set of M of these pre-steered temporal frequency snapshots, also known as Fourier vectors, $\mathbf{y}_k$, $k=k_1, \dots, k_1+M-1$, ($M \geq N$) arranged as the columns of a $N \times M$ data matrix. Each column of $\mathbf{Y}$ corresponds to one frequency bin in the FFT output. We form an estimate of the $N \times N$ steered covariance matrix (STCM), $\hat{\mathbf{R}}$, via

$$\hat{\mathbf{R}} = \mathbf{Y} \mathbf{Y}^H. \quad (2.1)$$

The $N \times 1$ STMV weight vector, $\hat{\mathbf{w}}$, is then computed via the familiar MVDR expression

$$\hat{\mathbf{w}} = \frac{\hat{\mathbf{R}}^{-1} \bar{\mathbf{1}}}{\bar{\mathbf{1}}^H \hat{\mathbf{R}}^{-1} \bar{\mathbf{1}}} \quad (2.2)$$

where $\bar{1}$ is a $N \times 1$ vector of ones which represents the steering vector appropriate to our pre-steered data. With the weight vector available, it is now possible to postulate two applications: (a) a NB application where the weight vector is applied to each of the Fourier (i.e. temporal frequency) vectors separately to give NB power estimates

$$|z_k|^2 = |\hat{\mathbf{w}}^H \mathbf{y}_k|^2, \quad k = k_1, \dots, k_1 + M - 1, \quad (2.3)$$

or (b) a WB application where the individual complex outputs $\{z_k\}$, $k = k_1, \dots, k_1 + M - 1$, are recombined into a time-series via an inverse DFT.

2.3 STMV-SA Beamforming

In practice two problems emerge in connection with the STMV method, both of them relating to the dimensionality of the STCM matrix. The first lies in the computational effort required for the matrix inversion in (2.2), roughly $O(N^3)$, which implies the computational loading increases sharply with the number of sensors, N . The second problem lies in the lack of stability of the weight vector estimate as the number of snapshot vectors (M) approaches the dimensionality (N) of the STCM matrix¹. This translates to an increase in the so-called MVDR misadjustment problem, which in turn can lead to a dramatic decrease in the available output SNR. Attempting to solve this problem by increasing the number of snapshots involved in estimating $\mathbf{R}$ leads to an over-widening of the processing bandwidth and/or to the use of multiple time-domain snapshots (with an appropriate "forgetting factor") with a concomitant increase in the effective response time of the system.

Both problems are addressed simultaneously in the partially-adaptive STMV-SA approach by invoking the MVDR process only on the outputs of a reduced number of subarrays (SA's), each performing simple "delay-and-sum" beamforming. If each SA comprises N_s contiguous sensors from the original array, then the output from each SA is simply the sum of the N_s corresponding pre-steered outputs from the original array.

Expressed mathematically, each $N \times 1$ pre-steered vector, $\mathbf{y}_k$, is replaced in the STMV-SA processor by a smaller ($N_a \times 1$) vector $\mathbf{y}_k^a$ given by something similar to the following example:

¹ More accurately, as the number of snapshots approaches the number of adaptive degrees of freedom, $N-1$

$$\mathbf{y}_k^a = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \mathbf{y}_k = \mathbf{K}^T \mathbf{y}_k. \quad (2.4)$$

In the above example we have $N=8$ sensors and $\mathbf{K}$ is a $N \times 3$ "subarray selection matrix", each column of which contains a contiguous grouping of $N_s = 4$ ones whose locations indicate the selection of the sensor members for the corresponding SA. As shown, there are three SA's ($N_a = 3$) and $\mathbf{y}_k^a$ is a 3×1 vector thus reducing the dimensionality of the ensuing adaptive stage.

The offset between adjacent groups (as shown above, offset = 2) controls the physical overlap between subarrays. Again in our example, if we had chosen offset = 4 then the SA's would be contiguous but with no overlap (i.e. no shared elements). In the latter case, with $N_s = 4$, offset = 4, and $N = 8$ we can only form $N_a = 2$ subarrays. With an offset of one, $N_s = 4$ and $N = 8$ we could form $N_a = 5$ SA's. In general, the number of subarrays is given by

$$N_a = \frac{(N - N_s)}{\text{offset}} + 1. \quad (2.5)$$

We will introduce other forms of the $\mathbf{K}$ matrix later. The original MVDR (or STMV) case is given by $N_s = 1$, offset = 1 and $N_a = N$.

The STMV-SA process simply requires that $\mathbf{y}_k$ in (2.1) be replaced by $\mathbf{y}_k^a$ in the formation of the covariance matrix estimate. This implies replacing the $N \times N$ STCM matrix estimate $\hat{\mathbf{R}}$ by the smaller $N_a \times N_a$ matrix $\hat{\mathbf{R}}^a$ where

$$\hat{\mathbf{R}}^a = \mathbf{K}^T \hat{\mathbf{R}} \mathbf{K}. \quad (2.6)$$

The $N_a \times 1$ MVDR weight vector is then given by

$$\hat{\mathbf{w}}^a = \frac{\hat{\mathbf{R}}^{-1} \bar{\mathbf{1}}}{\bar{\mathbf{1}}^H \hat{\mathbf{R}}^{-1} \bar{\mathbf{1}}} \quad (2.7)$$

where $\bar{\mathbf{I}}$ is understood to be $N_a \times 1$. For the k^{th} Fourier vector, the final output of the processor is $z_k = \hat{\mathbf{w}}^{all} \mathbf{y}_k^a = \hat{\mathbf{w}}^{all} \mathbf{K}^T \mathbf{y}_k$. Equivalently, the overall $N \times 1$ weight vector applied to the original (but pre-steered) data, $\mathbf{y}_k$, is

$$\hat{\mathbf{w}}_{sa} = \frac{\mathbf{K} \hat{\mathbf{w}}^a}{\bar{\mathbf{I}} \mathbf{K} \hat{\mathbf{w}}^a} \quad (2.8)$$

The denominator in (2.8) is a scaling factor necessary to yield true unity gain in the steering direction. We reiterate that if $N_s = 1$ and offset = 1 then $N_a = N$, $\mathbf{K}$ is the $N \times N$ identity matrix, and $\hat{\mathbf{w}}_{sa}$ in (2.8) is identical to $\hat{\mathbf{w}}$ in (2.2).

2.3.1 Impact of the structure of the selection matrix, $\mathbf{K}$

In order to examine the importance of the matrix $\mathbf{K}$, we must first establish a useful baseline scenario. For our purposes we assume that the field comprises (within our processing bandwidth):

(a) a plane-wave, tonal signal component (actual temporal frequency f_0 is immaterial) in the desired (pre-steered) direction at bearing θ_0 and with "in the bin" power, P_s . (See footnote 2). The corresponding steering vector (after pre-steering) is $\bar{\mathbf{I}}$.

(b) a plane-wave tonal interferer, of power P_i , at frequency $f_i (\neq f_0)$, to avoid the classic multipath-type problem associated with MVDR processing) and bearing θ_i . The N -element steering vector associated with this component (after pre-steering) is $\mathbf{e}_i$ whose n^{th} element is given by

$$\begin{aligned} e_i[n] &= \exp(j2\pi f_i n d (\sin(\theta_i) - \sin(\theta_0)) / c) \\ &= \exp(jn v_i) \end{aligned}$$

where v_i is understood to represent the effective spatial frequency of the interferer after the pre-steering shift, i.e. $v_i = 2\pi f_i d (\sin(\theta_i) - \sin(\theta_0)) / c$.

² The signal power is related to the time-domain amplitude, A , measured at the sensor by $P_s = (AL/2)^2$ where $L = F_s T$ is the number of points in the FFT process

(c) Self noise which is both spatially and temporally white with average power (per temporal frequency bin), P_n .³

Based on these assumptions we can write an expression for the infinite-data $N \times N$ STCM matrix $\mathbf{R}$ as

$$\mathbf{R} = P_s \bar{\mathbf{I}} \bar{\mathbf{I}}^H + P_i \mathbf{e}_i \mathbf{e}_i^H + M P_n \mathbf{I}_N \quad (2.9)$$

where M is the total number of Fourier vectors involved in the processing band and $\mathbf{I}_N$ is the $N \times N$ identity matrix. If the processing bandwidth is Δf , then $M = \Delta f T$. Note that (2.9) also assumes that the two tonals are each centred on a different frequency bin and hence each affects only a single Fourier vector. Equation (2.9) differs from the finite-data sample matrix estimate $\hat{\mathbf{R}}$ primarily in the third term which is truly valid only for $M \rightarrow \infty$. The impact of finite M will be examined in a later chapter.

We may write (2.9) as the sum of a signal covariance matrix, $\mathbf{R}_s$, an interference covariance matrix, $\mathbf{R}_i$, and a noise covariance matrix, $\mathbf{R}_n$ via

$$\mathbf{R} = \mathbf{R}_s + \mathbf{R}_i + M \mathbf{R}_n \quad (2.10)$$

with the obvious correspondence between the terms on the right hand sides of (2.9), (2.10).

Having chosen the $N \times N_a$ selection matrix, $\mathbf{K}$, then following steps (2.6) - (2.8) we find the overall $N \times 1$ weight vector for the STMV-SA method to be

$$\mathbf{w}_{sa} = \frac{\mathbf{K}(\mathbf{K}^T \mathbf{R} \mathbf{K})^{-1} \bar{\mathbf{I}}_{N_a}}{\bar{\mathbf{I}}_{N_a}^H (\mathbf{K}^T \mathbf{R} \mathbf{K})^{-1} \bar{\mathbf{I}}_{N_a}} \quad (2.11)$$

where the dimensionality of the $\bar{\mathbf{I}}$ vectors is explicitly subscripted. We remark again that the matrix inverses are of dimension $N_a \times N_a$.

³ P_n is related to the sensor-level time-domain real noise variance σ^2 by $L\sigma^2 = P_n$.

2.4 Some Infinite-data Simulation Results

2.4.1 NB Processing

Here we assume that the weight vector $\mathbf{w}_{\text{sa}}$ is to be used on each bin (i.e. each Fourier vector) separately as outlined in (2.3). Of interest to us is the degree to which our ability to detect the signal component is compromised by the weight vector's having to take into account the interference, albeit at a different frequency. We are also interested in the degree to which the same weight vector, when applied to the Fourier vector containing the interference, rejects the interference energy. Finally, we are interested to the extent to which the processing of a noise-only Fourier vector is compromised by the use of this same weight vector. Recall that in the usual case of NBMV, there would be a separate weight vector at each of the different frequencies across the band.

While this topic is described as "NB" it should firmly be grasped that the weight vectors (with the exception of the benchmark NBMV) are computed from a "WB" pre-steered covariance matrix.

For comparison purposes we shall examine the relative performance of:

(i) Hamming-shaded, unity gain, conventional beamformer (CB)

$$w_{CB}[n] = (1 - 0.85 \cos(2\pi n / N)) / N ; \quad (2.12)$$

(ii) Ideal NBMV at frequency of interest f_k ,

$$\mathbf{w}_{\text{NBMV}} = \frac{\mathbf{R}(f_k)^{-1} \bar{\mathbf{I}}}{\bar{\mathbf{I}}^H \mathbf{R}(f_k)^{-1} \bar{\mathbf{I}}} ; \quad (2.13)$$

(iii) STMV-SA1. Heavily overlapped STMV-SA with offset = 1 and a fixed number of subarrays $N_a = N - N_k + 1$.

$\mathbf{w}_{\text{sa}1}$ given by (2.11) with appropriate choice of matrix, $\mathbf{K}$.

(iv) STMV-SA2. STMV-SA with contiguous but non-overlapped subarrays (offset = N_s). N_a is chosen to match that of SA1 in (iii).

Note that for some choices of N , N_a , methods (iii) and (iv) may be mutually exclusive. Also note that ensuring N_a is the same in (iii) and (iv) ensures that the number of adaptive degrees of freedom ($N_a - 1$) is the same in both STMV-SA approaches, though the number of elements per subarray (N_s) may be much different. Finally, note that (iii), (iv) differ only in the choice of selection matrix, $\mathbf{K}$.

2.4.1.1 Processing of the signal component

(i) CB method

As a baseline we calculate the expected signal power output and the expected noise power output from the temporal snapshot (Fourier vector) containing the signal. The signal power is $\mathbf{w}_{CB}^H \mathbf{R}_s \mathbf{w}_{CB}$ ($= P_s$ by design) and the mean noise output power is $\mathbf{w}_{CB}^H \mathbf{R}_n \mathbf{w}_{CB}$ ($= P_n \|\mathbf{w}_{CB}\|^2$). The ratio of the two is the output SNR. The input SNR (measured "in the bin") is just P_s/P_n . The ratio of the SNR's is the array gain, AG, so we have

$$AG_{CB} = \frac{\mathbf{w}_{CB}^H \mathbf{R}_s \mathbf{w}_{CB}}{\mathbf{w}_{CB}^H \mathbf{R}_n \mathbf{w}_{CB}}. \quad (2.14)$$

It is well known for the Hamming shaded beamformer, AG_{CB} [in dB units] is given by $10 \log_{10}(N) - 1.3$ dB, or about 1.3dB worse than the optimum (unshaded) beamformer. It should be noted in this particular NB discussion, the interference plays no part in the CB calculations as it assumed to occur at some other frequency, i.e. in some other bin.

(ii) Ideal NBMV

Following the same approach we may compute the array gain, AG_{NBMV} , for a NB-MVDR beamformer at the signal frequency. It is of identical form to (2.14) with $\mathbf{w}_{CB}$ replaced by (2.13). Specifically, $\mathbf{w}_{CB}$ is replaced by

$$\mathbf{w}_{NB\Delta n} = \frac{(\mathbf{R}_s + \mathbf{R}_n)^{-1} \bar{\mathbf{I}}}{\bar{\mathbf{I}}^H (\mathbf{R}_s + \mathbf{R}_n)^{-1} \bar{\mathbf{I}}} \quad (2.15)$$

This beamformer is fully-adaptive with $N-1$ degrees of freedom. Again note that the interference term is irrelevant.

It is convenient to work with the array gain improvement (AGI) of the NBMV relative to the CB defined by

$$\text{AGI}(\text{NBMV}, \text{CB}) = \text{AG}_{\text{NBMV}} / \text{AG}_{\text{CB}} \quad (2.16a)$$

Recalling that our beamformers are all unity gain by design, $\mathbf{w}^H \mathbf{R}_s \mathbf{w} = P_s$, it is not difficult to show that the above may be written as

$$\text{AGI}(\text{NBMV}, \text{CB}) = \frac{\mathbf{w}_{\text{CB}}^H \mathbf{R}_n \mathbf{w}_{\text{CB}}}{\mathbf{w}_{\text{NB\Delta n}}^H \mathbf{R}_n \mathbf{w}_{\text{NB\Delta n}}} \quad (2.16b)$$

(iii) SA1. STMV-SA with offset = 1; N_d fixed.

We compute the array gain AG_{SA1} , from (2.14) but with a weight vector $\mathbf{w}_{\text{SA1}}$ given by (2.11). Again we compute an AGI value relative to the CB via

$$\text{AGI}(\text{SA1}, \text{CB}) = \text{AG}_{\text{SA1}} / \text{AG}_{\text{CB}} \quad (2.17)$$

(iv) SA2. STMV-SA with offset= N_s ; N_d fixed as above.

Again we compute an AGI value from

$$\text{AGI}(\text{SA2}, \text{CB}) = \text{AG}_{\text{SA2}} / \text{AG}_{\text{CB}} \quad (2.18)$$

Results

We employ the above expressions in connection with an $N=24$ element line-array with $P_s = 10$, $P_n = 1$, $P_i = \{0.1, 1, 10, 100, 1000\}$. For the STMV approaches we also choose $N_d = \{2, 4, 6\}$ implying $\{1, 3, 5\}$ degrees of freedom respectively. For

convenience, in all the following examples the number of temporal frequency snapshots M is set to $M = N$.

Fig. 2.1 shows the benchmark AGI(NBMV, CB) for the case where $P_i = 100$ (INR=20dB) along with the corresponding AGI(SA1, CB) where $N_a = 4$. In the latter we have subarray offset $= 1$ and $N_s = 21$ elements for each of the four subarrays. The subarrays are obviously heavily overlapped. The horizontal axis in Fig. 2.1 is the effective spatial frequency of the interferer $\nu_i = 2\pi f_d(\sin(\theta_i) - \sin(\theta_o)) / c$ which in general increases as the angular separation between the signal and the interferer increases. The figure shows that the major difference in AGI performance is in the vicinity of the mainlobe of the CB (low ν_i) where the SA1 is inferior by a few dB. Careful inspection of Fig. 2.1 outside the mainlobe region of the CB shows that both MVDR-based beamformers are better than the CB by about 1dB and are thus close to optimum in performance.

In Fig. 2.2 we have the corresponding result for AGI(SA2,CB) where again $N_a = 4$ but now offset $= N_s = 6$. In the vicinity of the mainlobe of the CB the SA2 performance is a little better than the SA1 case but there is evidence of a loss of AGI near, but not at, $\nu_i = \pi/3, 2\pi/3$ and π . Interestingly these correspond to the locations of grating lobes which might be expected of our set of non-overlapped SA's with an offset of 6. This is worthy of further discussion as follows:

Near the grating lobe positions ($\nu_i = p2\pi/N_s$, $|p|$ positive integer) the SA's outputs differ in phase by multiples of 2π , approximately. Alternatively stated, the $N_a \times 1$ interference output vector is nearly proportional to $\bar{1}_{N_a}$, precisely the steering vector used in the STMV-SA processors of (2.7) or (2.11). This means the SA2 processor will pass the interference from the region of the grating lobe as if it were actually signal. However, exactly at the grating lobe positions, the SA2 subarrays completely cancel the interference so the overall effect is found to each side of, but not at, the grating lobe positions. We see this effect most strongly on Fig. 2.2 near the grating lobe position $\nu_i = \pi/3$.

The results of Figs. 2.1 and 2.2 are typical of many other scenarios examined during the course of this work (but not plotted here). The SA1 method has inferior performance at low spatial frequencies (close interferers), the SA2 method has problems near its grating lobes.

In order to minimize the number of results to be plotted, it is useful to consider the relative performance of just the two SA methods. Fig. 2.3 shows $AGI(SA1, SA2) = AGI_{SA1}/AGI_{SA2}$ for the above scenario but for all five P_i values, $\{0.1, 1, 10, 100, 1000\}$ or $INR = \{-10, 0, 10, 20, 30\}$ dB. The approach taken to its interpretation is as follows:

In regions where $AGI(SA1, SA2) \sim 0$ dB both methods are performing close to the NBMV benchmark. If the AGI values dip much below 0 dB then the SA1 method is inferior, typically at low values of interference spatial frequencies; if the AGI values are above 0 dB then the SA2 is inferior, typically near its grating lobes. From Fig. 2.3 we can say that the multiple regions of inferior performance of the SA2 method, particularly at high INR, make it the inferior technique.

In Figs. 2.4 and 2.5 we plot the corresponding results for $N_a = \{2, 6\}$, respectively, and again our previous observations remain valid. The performance of the SA2 method near its grating lobe positions renders it inferior.

2.4.1.2 Processing at the interference frequency

By assumption, at the interference frequency, the snapshot (Fourier) vector contains only interference and noise contributions, no signal. The processing goal is to minimize the interference and noise power output from such a snapshot. We may thus define an "interference and noise improvement factor" (INI) to compare any two of our beamformers. For example, if we wish to compare the NBMV and the CB beamformers, we may write

$$IGI(NBMV, CB) = \frac{\mathbf{w}_{CB}^H (\mathbf{R}_i + \mathbf{R}_n) \mathbf{w}_{CB}}{\mathbf{w}_{NBMV}^H (\mathbf{R}_i + \mathbf{R}_n) \mathbf{w}_{NBMV}}. \quad (2.19)$$

As in (2.16) we note that this metric is independent of the signal level. The CB, SA1, SA2 weights are as computed earlier. However, the NBMV weights are different as they are now explicitly calculated at the interference frequency (see (2.13)).

Fig. 2.6 shows $INI(NBMV, CB)$ and $INI(SA1, CB)$ plotted against the effective spatial frequency of the interferer for the same scenario as Fig. 2.3. We note the somewhat superior performance of the NBMV method, as might be expected. In Fig. 2.7

we repeat the experiment but use the SA2 beamformer. Note the dramatic loss of performance for the SA2 beamformer near its grating lobe positions.

In Fig. 2.8 we use $INI(SA1, SA2)$ to compare the two SA beamformers in the manner of Fig. 2.3. We see that the SA2 is much inferior due to its grating lobe problems, especially at high INR. This effect is duplicated in Figs. 2.9, 2.10 which repeat Fig. 2.8 but use $N_a = \{2, 6\}$, respectively. Again we note the superiority of the SA1 processor.

Overall then, we can state that these experiments indicate that the SA1 processor with its heavily overlapped subarrays is superior in performance to the SA2 beamformer with non-overlapping subarrays and is inferior to the benchmark NBMV only for close interferers.

2.4.1.3 Noise-only processing

The remaining NB case to be explored is that where the output frequency of interest contains only noise. In this case our INI metric reduces to, say,

$$INI(NBMV, CB) = \frac{\mathbf{w}_{CB}^H(\mathbf{R}_n)\mathbf{w}_{CB}}{\mathbf{w}_{NBMV}^H(\mathbf{R}_n)\mathbf{w}_{NBMV}}. \quad (2.20)$$

We note that this is identical to the AGI definition in (2.16b) and, as the weight vectors are unchanged from those used in the signal frequency processing of section 2.4.1, then we have no need to repeat the calculations here. We may just substitute the label INI for AGI in Figs. 2.1-2.5 and repeat the discussion.

2.4.1.4 Comment on NB Processing in [2-2]

In the degenerate case of $N_s = 1$, $\text{offset} = 1$ and $N_a = N$ then both SA1 and SA2 reduce to the fully adaptive STMV with $N-1$ degrees of freedom. This fact is only relevant in that it relates to the so-called NB processor of Dhanatawari [2-2, 2-3] who calculates the fully-adaptive STMV weight vector according to (in our notation)

$$\mathbf{w} = \frac{\mathbf{R}^{-1}\mathbf{e}_c}{\mathbf{e}_c^H \mathbf{R}^{-1} \mathbf{e}_c} \quad (2.21)$$

where $\mathbf{e}_c$ is the NB steering vector at the centre frequency of the processing band in the chosen steering direction. Clearly, according to our analysis above, this is a rather strange choice of steering vector; the obvious choice (recall that $\mathbf{R}$ is pre-steered) is $\mathbf{e}_c = \bar{\mathbf{T}}$. Next in [2-2], the weight vector of (2.21) is modified to be "NB" and the modification is a function of the NB frequency of interest. This again is contrary to the spirit of STMV processing where the same weight vector is used across all the frequency bins.

It may come as little surprise, then, that the NB processor of [2-2] was deemed to be a failure. However, the degree of failure is interesting. In the presence of a single CW tonal located near broadside, there was a strong, almost uniform response at the tonal frequency *across all bearings*. So marked is this effect [2-2, p55] that it begs to be discussed⁴ but it is cursorily dismissed as an "artifact" due to "problems in extrapolating the NB weight vectors". However, one is left with the feeling that this result coloured the author's perception of the utility of any NB processing based on the STMV approach - such artifacts will always be present and have to be "suppressed" by the STMV-SA beamformer and its variants. This is unfortunate as the results presented in the current work show that useful and computationally efficient processing in the NB mode is available, and is entirely free of the type of artifact generated in [2-2]. There remain the problems of multi-path scenarios and finite-data issues, but these are altogether different matters.

2.4.1.5 Summary of NB Processing

The above experiments have been concerned with infinite-data, NB processing with a uniform line array in a non-multipath environment. They have demonstrated that the STMV-SA1 method with heavily overlapped subarrays with an inter-array offset of one sensor is superior to the STMV-SA2 with non-overlapped but contiguous subarrays. The latter is bedeviled by potentially serious problems related to grating-lobe structures. The results assume that the same number of adaptive degrees of freedom is used in both methods and that practical performance is approximated by the idealised model used in the analysis. Finite-data and multipath effects will be examined later.

⁴ It would seem that this type of result is indeed possible if the source is located near broadside and $\mathbf{e}_c$ is replaced by its complex conjugate in (2.21)

2.4.2 Wideband Processing

Here we are interested in constructing a time-series output from the beamformers (BF's) as mentioned in Section 2.2. Ideally it should be identical to that emanating from the source (if present) in the steering direction with the effect of noise and interference minimized.

Notwithstanding the fact that our output is now a time-series, we can still quantify performance on an SNR-type basis and the necessary calculations can still be performed in the frequency domain. We shall focus on the SA1 and SA2 beamformers and use the Hamming-shaded CB and the fully-adaptive STMV as references. The weights of the first three BF's are calculated as detailed above. However, the STMV weights are given by (2.2) or equivalently by setting $\mathbf{K}$ in (2.11) to be the $N \times N$ identity matrix. Again it is important to remember that these weights are computed using all the data and that for each beamformer a single set of weights is used across all the bins.

2.4.2.1 WB Performance metrics

We again use the signal tonal plus interference tonal plus self-noise model as used in the foregoing NB processing experiments. Let us consider first the CB beamformer. By Parseval's theorem, the signal power in the output time-series is again $\mathbf{w}_{CB}^H \mathbf{R}_s \mathbf{w}_{CB} = P_s$. The expected noise plus interference output power is $\mathbf{w}_{CB}^H (\mathbf{R}_i + M\mathbf{R}_n) \mathbf{w}_{CB}$ hence a useful output metric is the signal to interference plus noise ratio, SINR, given by

$$SINR_{CB} = \frac{\mathbf{w}_{CB}^H (\mathbf{R}_s) \mathbf{w}_{CB}}{\mathbf{w}_{CB}^H (\mathbf{R}_i + M\mathbf{R}_n) \mathbf{w}_{CB}} \quad (2.22)$$

Note the appearance of M , the number of FFT bins in the processing band, in the denominator. It reflects the fact that all M bins (or temporal frequency snapshots, $\mathbf{y}_k$) contribute to the overall noise output. With the appropriate choice of weight vector, (2.22) is used as the basis for all SINR calculations.

As previously, we can compare any two beamformers via the AGI figure which is simply the ratio of the corresponding SNIR's. Given the similarity of the NB and WB metrics (they differ only in the factor M), it should not be too surprising that the WB results are largely consonant with the NB ones. The results are shown in Figs. 2.11-2.15.

They are so similar to their counterparts, Figs. 2.1 to 2.5, or Figs. 2.6 to 2.10 that no further discussion is warranted.

2.5 The Multipath Problem

We now take a brief look at the multipath situation. This scenario, where the (desired) signal and the interference are highly correlated, has long been recognised as troublesome for MVDR-type processing as the beamformer uses information gleaned from the interference to cancel both the interference (a desirable effect) and the signal itself (highly undesirable). In our STCM version we have only a single temporal frequency snapshot (Fourier vector) for each FFT bin (recall we use a single WB time-series observation), hence any arrivals sharing the same FFT frequency bin will appear correlated to the SA beamformers, even though they may actually be uncorrelated. In the more usual case of multiple (time-domain) snapshots, the signal and interference contributions (may) become decorrelated through the implicit averaging action involved in forming the covariance matrix estimate.

In order to set up a performance measure for the multipath situation, we will use the following AGI definition:

$$AGI(SA1, CB) = \frac{\mathbf{w}_{SA1}^H(\mathbf{R}_s) \mathbf{w}_{SA1}}{\mathbf{w}_{SA1}^H(\mathbf{R}_n) \mathbf{w}_{SA1}} \frac{\mathbf{w}_{CB}^H(\mathbf{R}_n) \mathbf{w}_{CB}}{\mathbf{w}_{CB}^H(\mathbf{R}_s) \mathbf{w}_{CB}} \quad (2.23)$$

where $\mathbf{R}_s$ is

$$\mathbf{R}_s = \left[\sqrt{P_s} \mathbf{e}_0 + \sqrt{P_i} \mathbf{e}_i \right] \left[\sqrt{P_s} \mathbf{e}_0 + \sqrt{P_i} \mathbf{e}_i \right]^H \quad (2.24)$$

and $\mathbf{e}_0 = \bar{\mathbf{I}}$. Equation (2.24) reflects the fact that the (tonal) signal and interference are assumed to occur at the same frequency. They will thus exist in the same bin and will always appear to be fully correlated to our beamformers. In essence, (2.23) is the ratio of two SNR's, each of which is the ratio of "signal plus interference" output power to noise output power. The SA1, SA2 adaptive weights are computed from the WB process of (2.10), (2.11) with $\mathbf{R}_i$ set to zero.

2.5.1 Results

As before, we use $N=24$, $P_s=10$, $P_n=1$, $N_a=4$ and we examine the performance of the SA1 and SA2 partially-adaptive beamformers as a function of the interference level, P_i , and the interference location.

In Fig. 2.16 we have $AGI(SA1, CB)$ for $N_a=4$ and $P_i = \{0.1, 1, 10, 100, 1000\}$ or $INR = \{-10, 0, 10, 20, 30\}$ dB plotted against the effective spatial frequency, v_i , of the interference. The result is a little complicated and is deserving of some discussion, as follows.

We start by observing that increasing v_i is tantamount to increasing the angular separation between the signal and interference. The Hamming-shaded CB mainlobe-width extends to $v_i = 0.5$, approximately. Beyond this we can expect the CB sidelobes to eliminate most of the interference power, leaving the signal contribution unaffected. For $INR = -10$ dB we may assume that the interference power is too low (relative to the background noise) for the adaptive SA1 processor to use it for signal cancellation and hence at this low INR level (upper curve in Fig. 2.16) the two beamformers will have comparable performance. Indeed we see that the SA1 performance exceeds that of the CB by about 1dB as might be expected due to the CB's shading.

As we increase the INR, a significant scalloping effect is observed in Fig. 2.16. The peaks of the scalloping are quite narrow and are observed to fall at those interference spatial frequencies which correspond to nulls in the SA1 subarray beampatterns, i.e. at $v_i = 2\pi p/N_s$, $|p|$ positive integer. In this application, the SA1 BF has $N_s = 21$. At these locations, no interference information passes through the subarrays and hence no signal cancellation is possible and again the two BF's have similar performance (outside of the CB's mainlobe region). Between these locations, however, the SA1 beamformer will attempt signal (and interference) cancellation so that the AGI curves dip appreciably, showing that the SA1 power output is being reduced relative to the CB. It can easily be envisaged that there is a concomitant SNR reduction for the SA1 beamformer in these dips.

We now move on to examine the SA2 performance in the presence of signal and interference co-located in frequency. In Fig. 2.17 we re-run the experiment of Fig. 2.16

but use the SA2 beamformer where there is no subarray overlap. Again the results are worthy of interpretation in the light of the SA1 discussion:

In Fig. 2.17 we again observe a gross scalloping effect but now our subarrays are shorter ($N_s = 6$) and hence we expect scalloping peaks at spatial frequencies, $2\pi p/N_s = \pi/3, 2\pi/3, \pi$. These peaks are indeed visible on Fig. 2.17. Between these locations there are again regions of significant and undesirable signal cancellation, represented as deep dips in the AGI curves. Comparison of the dips in the two figures indicates that those of the SA2 beamformer are somewhat shallower.

2.5.2 Summary

This brief look at the problems presented by having signal and interference contributions co-located in frequency has shown that the SA1 and SA2 beamformers both share the same overall trait: If the interference happens to fall at a null of a subarray, no harmful signal cancellation occurs. Otherwise significant signal cancellation may be expected with a corresponding reduction in output SNR. The effect is more severe as the INR level increases. It should be noted that the signal and interference do not have to be genuinely correlated; they must just share the same frequency bin. If a distinction can be made, it would be to observe that the signal cancellation effect is more severe in the SA1 method.

Finally, it might be mentioned that it is possible for an unwary experimenter to randomly pick an interference location where no signal cancellation occurs and be misled into thinking that STMV-SA methods are generally impervious to signal cancellation effects.

2.6 Overall Conclusion

It has been demonstrated, in both the so-called NB and WB processing modes, that the partially-adaptive STMV-SA method of choice when dealing with non-multipath, tonal data is the SA1 beamformer with heavily overlapped subarrays. It does not suffer the loss of performance observed in the non-overlapped SA2 beamformer caused by grating-lobe effects. Its (non-multipath) performance is close to the benchmark full-adaptive case, except for interferers close to the chosen steering direction (within a conventional beamwidth, say) where it may be several dB inferior.

As expected, in the multi-path scenario, here implying simply that the signal and interference share the same frequency bin, both STMV-SA methods are heavily troubled by signal cancellation effects. In this case the SA2 seems slightly less affected.

Even with the SA1 beamformer chosen as the better processor overall, there remains the fundamental question: Given all the real-world problems associated with adaptive beamforming, is SA1 really a better choice than a non-adaptive BF like the Hamming shaded CB. From our "in the mean" results presented in this chapter, it seems that the SA1 BF outperforms the CB only in two situations: (1) if there are close interferences, within the mainlobe of the CB, say; (2) if there are really strong interferences ($\text{INR} > 30\text{dB}$). In all other circumstances, and taking real-world problems into account, the CB would appear to remain competitive. We move on to examine the impact of some of these real-world problems in the next chapters.

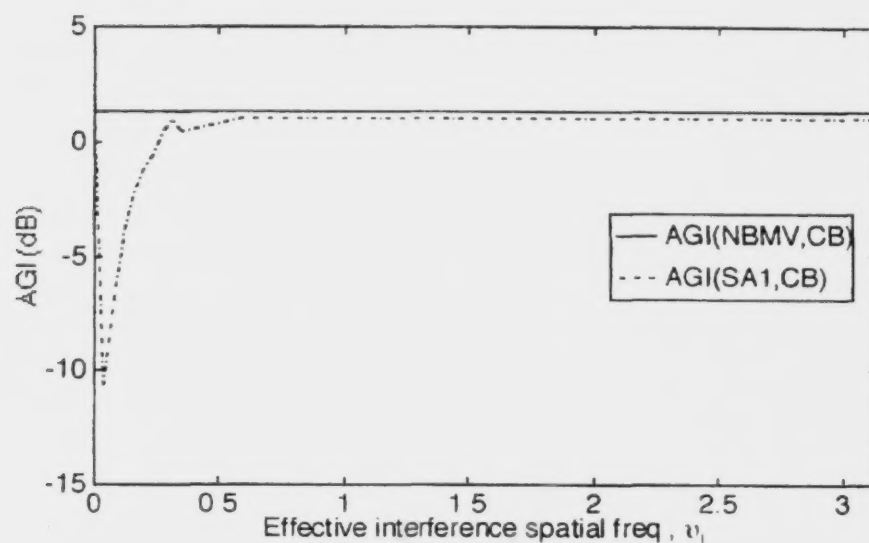


Fig. 2.1 AGI curves at signal frequency. $N=24$, $N_a=4$, $INR=20\text{dB}$. Note the inferior performance of the SA1 processor at low values of v_i .

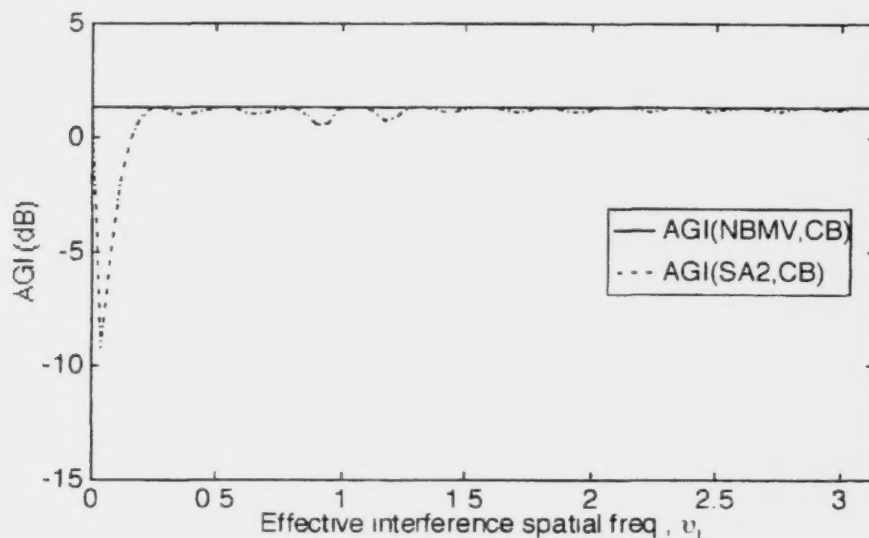


Fig. 2.2. As 2.1 but using the SA2 processor. Note the slight evidence of inferior performance near $v_i=1$.

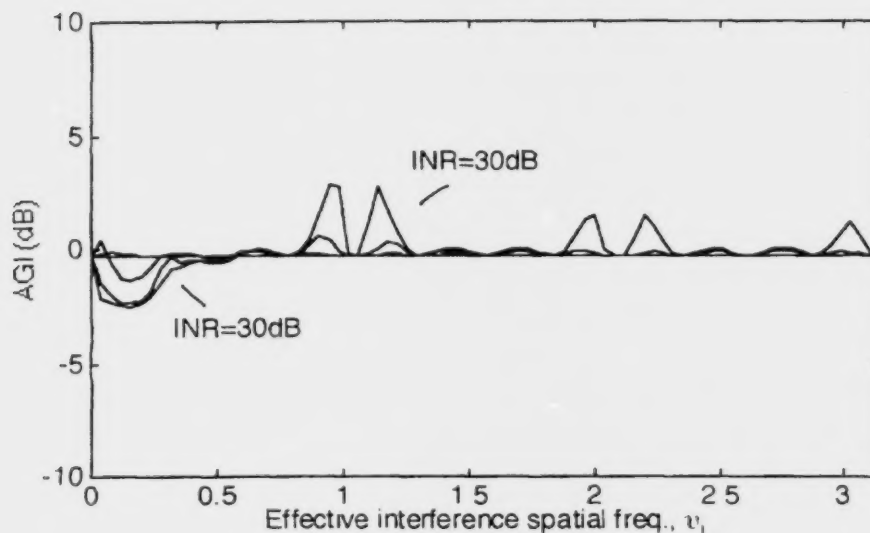


Fig. 2.3 AGI(SA1,SA2) for $N=24$, $N_a=4$, $\text{INR} = \{-10, 0, 10, 20, 30\}$ dB. The SA1 has inferior performance at low v_i . SA2 has inferior performance near $v_i = \pi/3, 2\pi/3, \pi$.

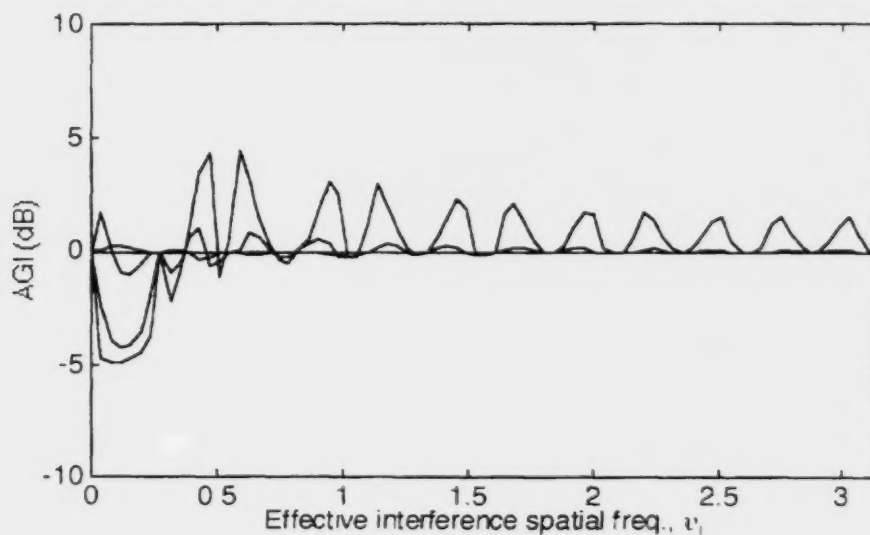


Fig. 2.4. AGI(SA1,SA2). As Fig. 2.3 but for $N_a = 2$. Note SA2 problems at $n\pi/6$.

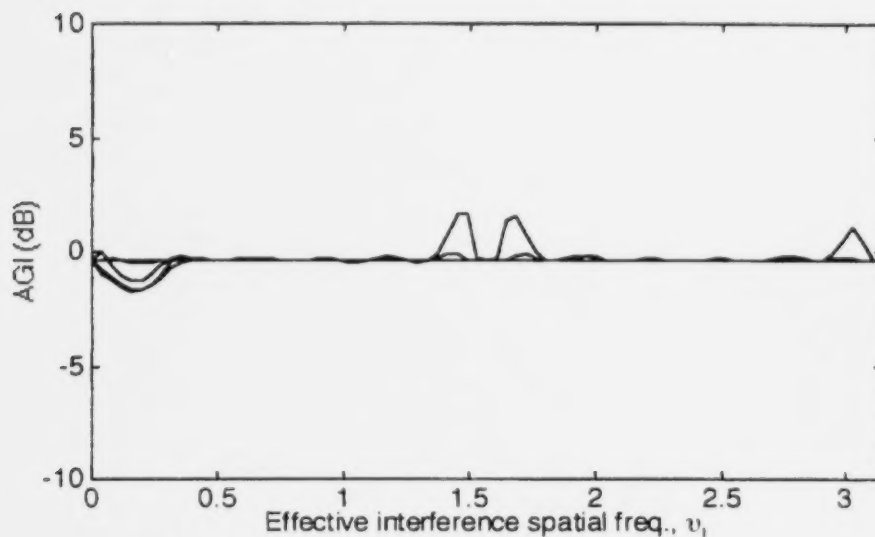


Fig 2.5 AGI(SA1, SA2) As Fig. 2.2, 2.3 but for $N_a = 6$. Note SA2 problems near $n\pi/2$

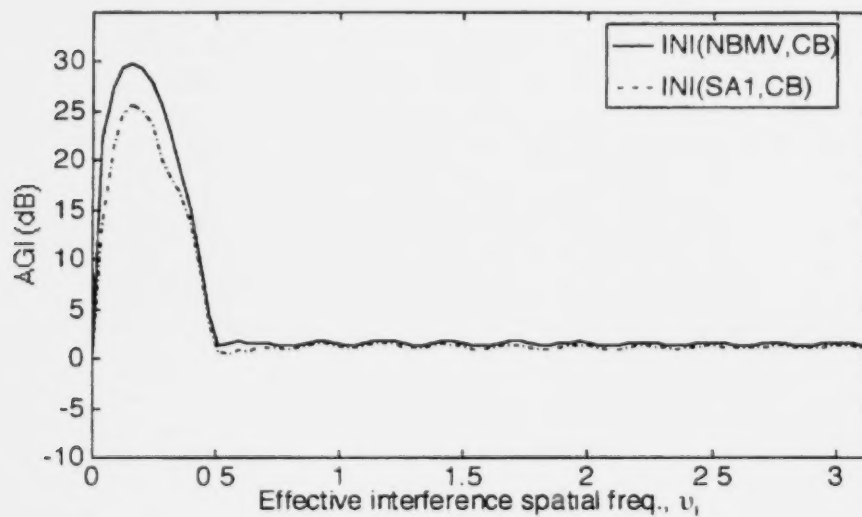


Fig 2.6 INI curves at interference frequency. $N=24$, $N_a=4$, $INR=20$ dB

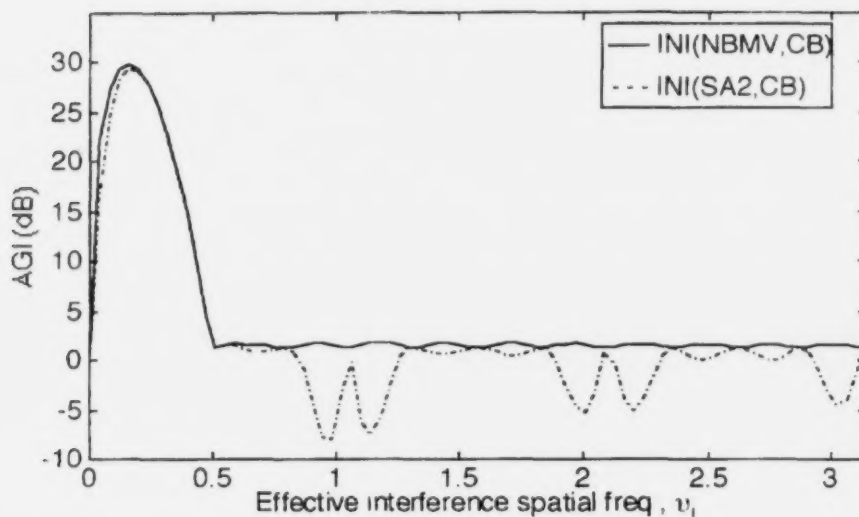


Fig. 2.7 INI curves. As Fig. 2.6 but with SA2 processor.

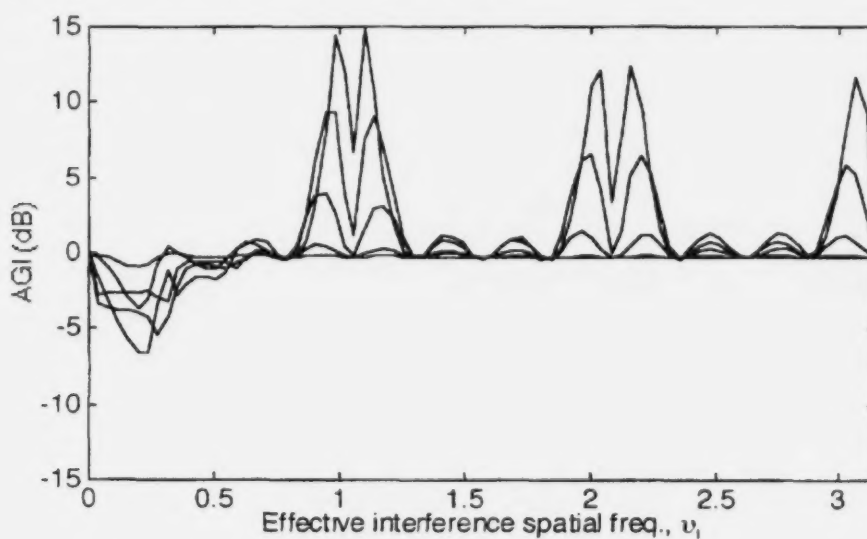


Fig 2.8 INI(SA1, SA2) $N=24$, $N_a=4$ and $INR = \{-10, 0, 10, 20, 30\}$ dB.

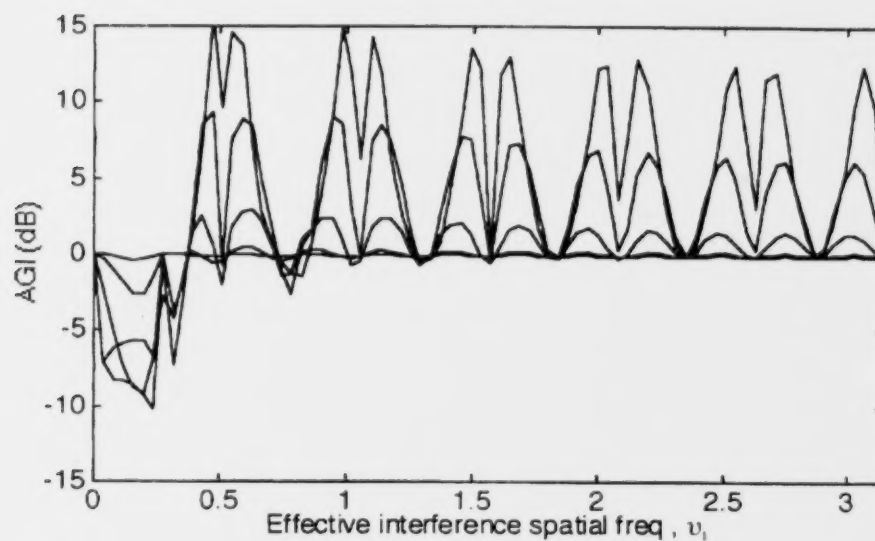


Fig 2.9 INI(SA1, SA2) curves. As Fig. 2.8 but with $N_d = 2$.

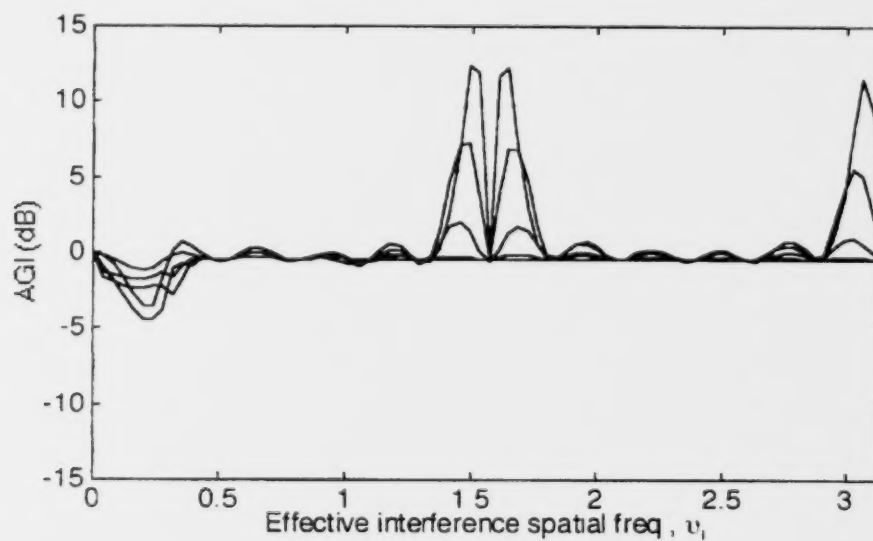


Fig 2.10 INI(SA1, SA2) curves. As Fig. 2.8 but with $N_d = 6$.

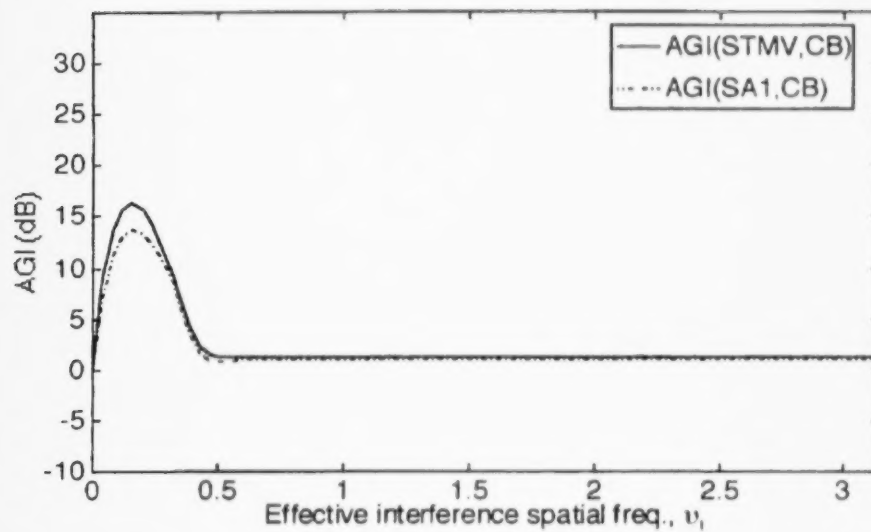


Fig 2.11 WB AGI curves $N_a=4$, $\text{INR}=20\text{dB}$

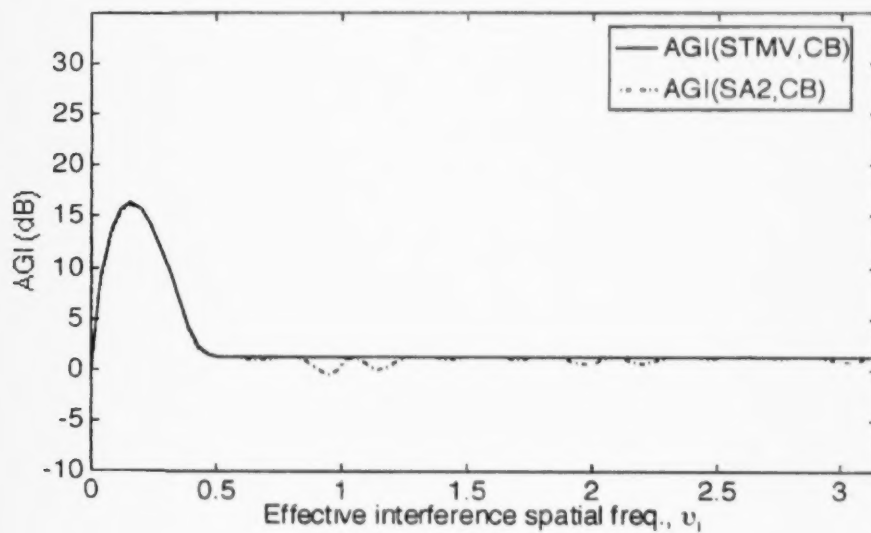


Fig 2.12 WB-AGI curves, $N_a=4$, $\text{INR}=20\text{dB}$.

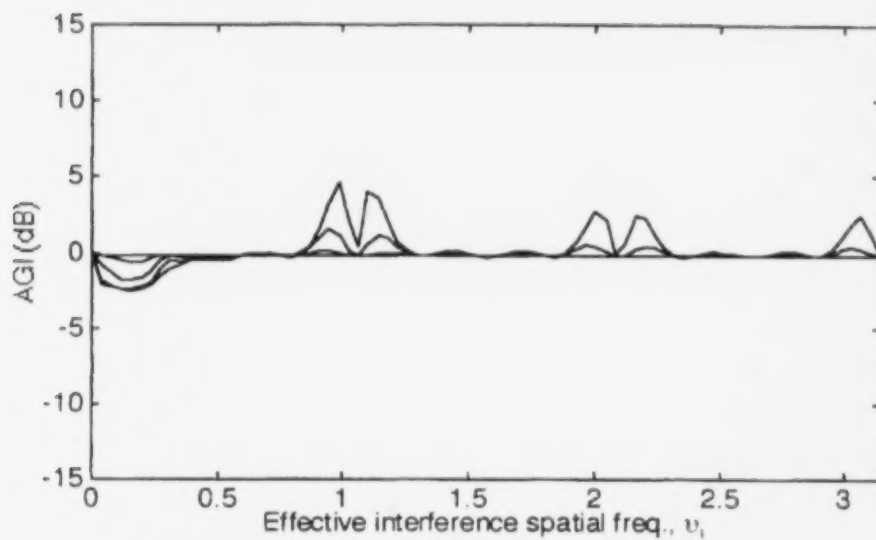


Fig. 2.13 WB AGI(SA1, SA2) curves. $N_d=4$. $\text{INR} = \{-10, 0, 10, 20, 30\text{dB}\}$

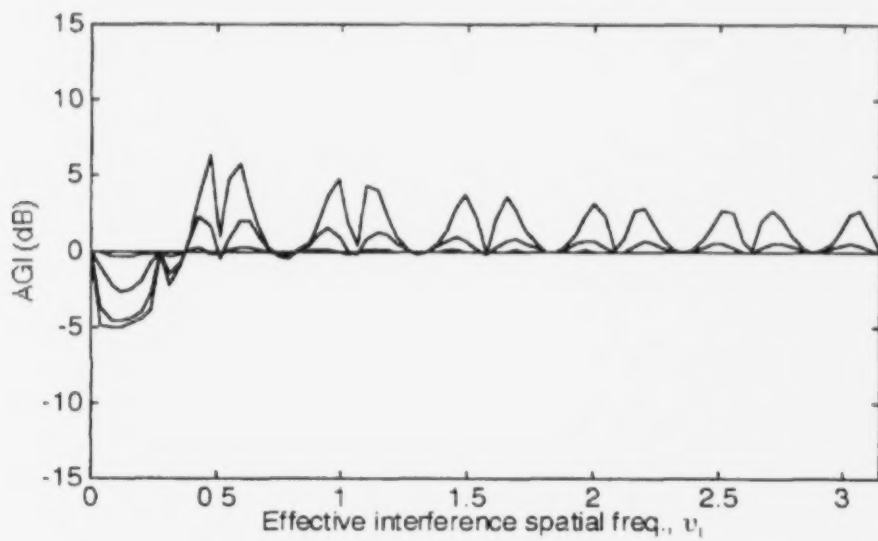


Fig. 2.14 WB AGI(SA1, SA2) curves. As Fig. 2.13 but with $N_d=2$

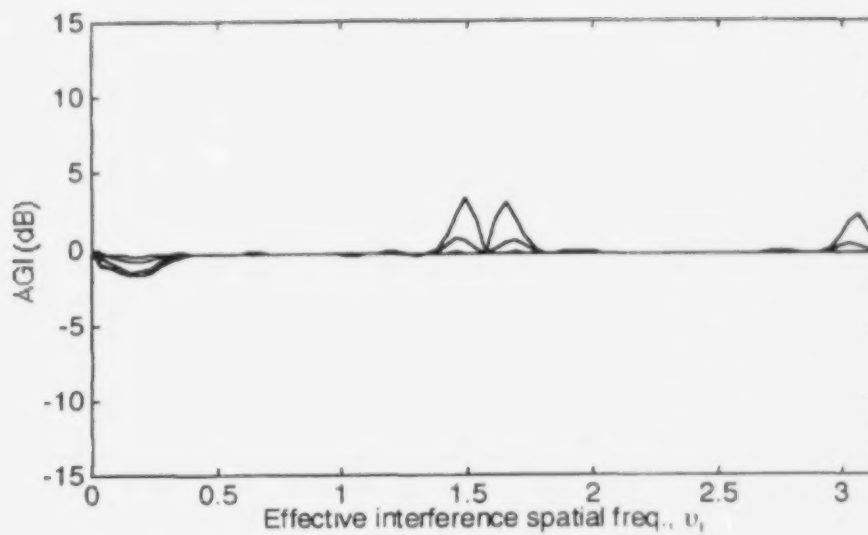


Fig 2.15. WB AGI(SA1, SA2) curves. As Fig. 2.13 but with $N_a=6$.

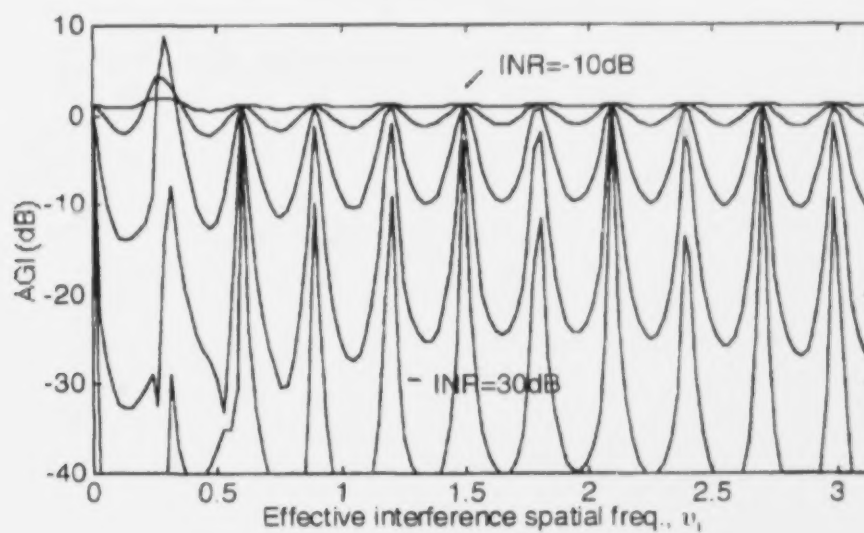


Fig. 2.16 Multipath Performance. AGI(SA1, CB). $N=24$, $N_a=4$, $N_s=21$

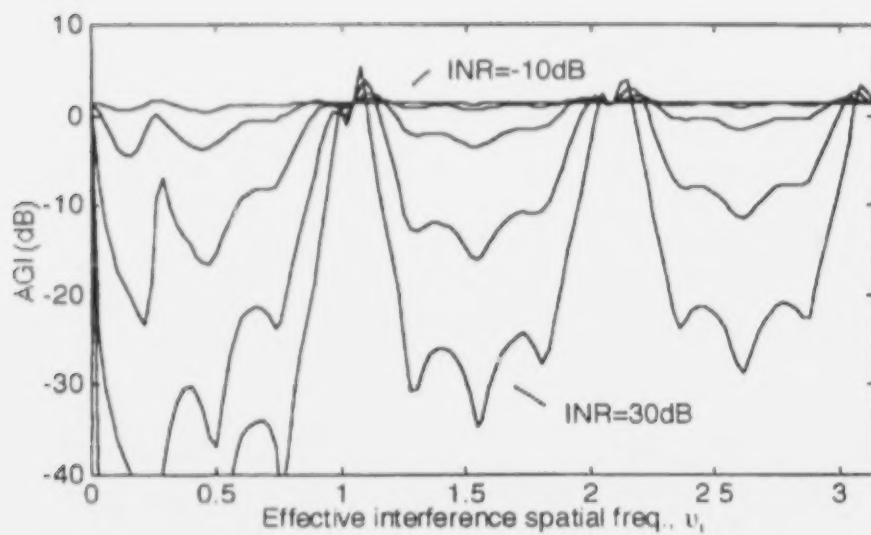


Fig. 2.17 Multipath Performance. AGI(SA2, CB). $N=24$, $N_d=4$, $N_s=6$.

3. FURTHER INFINITE-DATA RESULTS IN STMV-SA BEAMFORMING: SENSOR CHANNEL IMPERFECTIONS AND COHERENCE EFFECTS

3.1 Introduction

This chapter is essentially an extension to the previous chapter. It continues to deal with the performance of the STMV-SA1 beamformer in the absence of finite-data effects, i.e. we work with ideal covariance matrices and ignore the misadjustment phenomenon. However, we do start to further our investigation, begun in the last chapter with the multi-path discussion, into the STMV-SA1 performance in non-ideal scenarios, in particular where there are missing sensors and/or sensor perturbations. The difference in the two latter phenomena lies in the fact that in the one case we assume that we know which sensors are missing but in the other we do not have any *a priori* knowledge of the random sensor perturbations. In the former the MVDR-based beamformer can deal well with the problem; in the latter it may suffer a considerable SNR penalty. Both of these issues have practical relevance to DREA's efforts. We also look briefly at the impact of the coherence length of the impinging wavefronts.

In the case of missing sensors, the most important (and unexpected) result is that the STMV-SA1 beamformer with heavily overlapped sub-arrays, the preferred SA method from Chapter 2 and the focus of considerable recent study by DREA, is subject to a significant performance penalty associated with plane-wave interferers emanating from directions corresponding to nulls in the beampatterns of the "delay-and-sum" subarray(s). However, a simple remedy is shown to be available.

In the case of sensor perturbations, or misalignment, we demonstrate that in comparison to a Hamming-shaded CB, there may or may not be a performance gain. It depends primarily on the strength of the desired signal and the degree of sensor misalignment. In both matters, the number of adaptive degrees of freedom (and hence the number of subarrays) is relatively unimportant.

Finally, in our brief study of the impact of coherence length we demonstrate that providing the SNR of the desired signal is not too high, the SA1 beamformer remains competitive with the CB even in the presence of strong interference.

3.2 Missing Sensors.

3.2.1 Single Missing Sensor.

We initially assume our otherwise uniform line-array has a single sensor "missing". If this is sensor # n , then we say that the signal from the n^{th} sensor is zero. We shall adopt the same model as earlier where we have a desired tonal, planewave signal in the pre-steered direction at frequency, f_0 , and a tonal, planewave interferer at some other, well-separated, frequency, f_1 , with effective spatial frequency v_1 . We also continue to focus on the STMV-SA1 beamformer with identical SA's and a relative offset of one sensor position.

Let us consider the impact of an interferer at a null position of the subarray(s) of length N_s , i.e. at $v_1 = 2p\pi/N_s$, (p integer). If the SA is intact, the "delay-and-sum" SA interference output is zero, but if it contains the missing sensor, # n , then, by superposition, the interference output is just $-y_i[n]$, the negative of the interference signal at the n^{th} sensor. If the n^{th} sensor is common to all the SA's, which implies it is not too near either end of the full array, then we have exactly this same interference output from each of the subarrays. The subsequent MVDR process treats this as "signal" (for this is indeed the rationale behind pre-steering) and there is no MVDR interference rejection whatsoever. This in turn severely compromises the performance of the beamformer in NB mode at frequency f_1 . To a lesser extent it affects the performance at the signal frequency f_0 due to MVDR weight increases near these angles, which in turn increase the background noise.

It should be borne in mind that the CB beamformer's beampattern is also considerably affected by missing sensors so we shall compare the performance of the STMV-SA1 beamformer with that of the corresponding Hamming-shaded CB in the same environment.

In Fig. 3.1 we have the INI performance at frequency f_1 of the SA1 beamformer versus the CB beamformer (both adjusted for unity signal gain) in the case of a nominal $N=24$ sensor line-array with $N_a = 4$ subarrays, each of length $N_s=21$ as in Chapter 2. In this example, sensor #7 is missing. There is a weak signal with $\text{SNR} = -10\text{dB}$ ¹. The

¹ The results in this section were observed to be largely independent of SNR

effective spatial frequency of the interference is varied from 0 to π and the INR is varied from -10 dB to +30 dB in 10 dB steps.

In the $INI(SA1, CB)$ curves of Fig. 3.1 we see that for weak INR (lowest curve) the beamformers have similar performance, as expected. At higher INR's the adaptive SA1 beamformer has a gain of around 20dB (more at lower v_1 values), showing good interference rejection, except near regions given by $2\pi m/N_s$ where we see sharp nulls in the curves implying no improvement by the SA1 over the CB, whose interference rejection capability is itself compromised by the missing sensor.

The rationale for the nulls has been discussed above. If we pursue the notion that the problem is caused by identical SA performance, there is an obvious solution: unbalance the subarrays². In this case we might make three of the SA's of length 21, but the fourth we might make of length 22. In this case the SA responses to the interference are not identical and the difficulty is overcome. We see the results of this modified SA1 processing (known as STMV-SA1M or simply SA1M) in Fig. 3.2 which is a repeat of the experiment of Fig. 3.1 but with the modified subarrays. We see immediately that the problematic deep nulls have been largely eliminated. There remain some shallower dips in the same locations.

If we make the comparison of the SA1M beamformer applied to the array with the missing sensor versus an ideal CB with no missing sensor (CB0) then Fig. 3.3 ensues. We note that for strong INR the SA1M performance is generally better than CB0, but at modest INR's the SA1M performance occasionally dips 2dB or so lower than the CB0. Overall, however, we have demonstrated that the SA1M beamformer with three degrees-of-freedom (DOF = 3) is performing about as well as a fully-adaptive MVDR on a perfect array. This is to be expected, as the missing sensor information is made available to the MVDR process.

3.2.2 Multiple Missing Sensors

The situation with multiple missing sensors is very little different to the above. In Fig. 3.4 we have curves of $INI(SA1M, CB)$ for the situation where sensors { 7, 11, 17 } are

² Mathematical details are shown in the Appendix at the end of this chapter

missing. Again we see a notable improvement in performance of the SA1M beamformer over the CB, primarily due to its enhanced noise rejection capabilities. The improvement increases with INR.

If we choose to beamform the signal bin data (rather than the interference bin data as above), then Fig. 3.5 ensues. Again we have sensors {7, 11, 17} missing. The dips in the AGI(SA1M, CB) curves are caused by local increases in the weight norm which are usually a sign that the adaptive beamformer is finding it difficult to reject the corresponding interference. It should be stressed that the interference is not at the signal frequency and hence it will have no impact whatsoever on the CB results.

Though not shown here for space reasons, other experiments with other missing sensor combinations and different degrees of freedom gave substantially the same type of results. Hence we can say the SA1M (but not the SA1) beamformer is a computationally efficient adaptive beamformer which can deal well with missing sensors. At the interference frequency its performance generally exceeds that of the CB applied to the same array. This is due to its virtually undiminished ability to cancel interference. In this scenario then the adaptive method outperforms the non-adaptive method.

3.3 Sensor Perturbations

In the previous section, we saw that the SA1M adaptive beamformer deals well with a modified array. This is primarily because the information about the missing sensors is easily incorporated into the MVDR processing (see Section 3.7). The situation where the sensor "modifications" are not known to the processor is much different - it leads to the phenomenon of signal cancellation and, depending on the magnitude of the perturbations, greatly reduced output SNR.

We start with the SA1³ processor applied to the $N = 24$ sensor array to which random gain variations have been applied to the sensor output. If the nominal gain is G then the actual gain of the n^{th} sensor is $G(1+\delta_n)$ where δ_n is a real, zero-mean Gaussian random variate with variance σ . The variations are uncorrelated between sensors. This is a simple but effective model for examining the more general amplitude and phase perturbations. We saw in a previous contract report [3-1] that both amplitude and phase errors (if sufficiently modest) can be lumped together in the variance, σ . We define the

³We could have used the SA1M processor instead; the results are very similar.

perturbation error or deviation ratio (DVR) as $10\log(\sigma)$. In our example the standard deviation of the perturbations is $\sigma^{1/2} = 0.1$ and hence $\text{DVR} = -20\text{dB}$.

In Fig. 3.6 we plot the AGI(SA1,CB) curves which describe the performance of the SA1 processor versus the CB processor both applied to the perturbed array at the signal frequency. There is a single interferer of $\text{INR} = -20\text{ dB}$, a negligibly low figure. The input SNR is adjusted from 0 to 30 dB in 10 dB steps. The output SNR for the CB is not expected to be much affected and hence the curves reflect a deterioration in performance of the SA1 due primarily to signal cancellation effects linked to the sensor perturbations interacting with the high values of SNR. We see that there is a sort of threshold very roughly near $\text{SNR} = 20\text{dB}$ below which there is little signal cancellation (and the two beamformers offer similar performance) and above which there is a truly significant performance loss for the SA1 (of some 30dB in Fig. 3.6 where $\text{SNR} = 30\text{dB}$). In general, it seems that the threshold occurs near $\text{SNR} = -\text{DVR}$ which is consonant with the earlier report [3-1], remembering that we here have a WB-STCM matrix with signal energy only in one bin. The raggedness of the curves reflects the fact that the sensor perturbation realisation is deliberately changed for each different v_i and each SNR.

If we now beamform at the interference frequency we get the INI curves of Fig. 3.7 where again we see a significant loss in performance for the SA1 beamformer at higher SNR's. In this case there is no signal cancellation effect at play (recall we are processing at f_i but using the same weights as previously); however the increased weight norm associated with the signal cancellation phenomenon increases the background noise level by up to 20 dB in the non-signal bins.

Increasing the interference INR to 30 dB yields the AGI(SA1, CB) curves of Fig. 3.8. In comparison to Fig. 3.6 we see there is little interference effect, apart from at very low values of spatial frequency, v_i which corresponds to close-in interference. However, moving to the interference bin produces the INI(SA1, CB) curves of Fig. 3.9 where we see quite large differences in comparison to Fig. 3.7. For remote interference ($v_i > 0.5$) we see that at low SNR there is an INI gain of about 10dB for the SA1 beamformer. This is due to its ability to null the interference more successfully than the (perturbed) CB. However as the SNR grows, the SA1 introduces signal cancellation which increases the weight norm and hence the background noise level rises. Thus at higher SNR, the overall INI performance of the SA1 is inferior. For close interferences, the SA1 INI performance remains superior.

We should note that these results were observed to vary little with the number of degrees of freedom (here, $\text{DOF} = 3$).

3.4 Missing Sensors and Sensor Perturbations

We now give a demonstration of the performance of the SAIM beamformer in the presence of both sensor perturbations and missing sensors. We again set $\text{DVR} = -20\text{dB}$ and have sensors $\{7, 11, 17\}$ missing. We shall compare the results against the ideal CB benchmark (CB0) where there are no perturbations or missing sensors. We again have a $N=24$ sensor array, $N_s = 21$ (or 22 see above remarks on SAIM), $N_a = 4$ (so $\text{DOF} = 3$), $\text{SNR} = \{0, 10, 20, 30\}\text{dB}$, $\text{INR} = 30\text{dB}$, new sensor perturbations for each v_i and each SNR.

In Fig. 3.10 we have the $\text{AGI}(\text{SAIM}, \text{CB0})$ results at the signal frequency. The result is very similar to Fig. 3.8 where there were sensor perturbations only. The impact of the missing sensors is negligible. At higher SNR's the SAI suffers from both signal cancellation and a concomitant increase in background noise due to the increase in the weight norm. At the interference frequency we have the $\text{INI}(\text{SAIM}, \text{CB0})$ results of Fig. 3.11. At low SNR the curves show that we have performance that is similar to that for an unperturbed array. At higher SNR ($\geq 20\text{dB}$, or so) the increase in weight norm associated with the signal cancellation effect impacts the SAIM performance. However, we must stress that for low/modest SNR the performance is still better than that of the CB on the *perturbed* array.

3.5 Coherence effects

Finally, we turn our attention to the problem of the lack of spatial coherence in the signal and interference(s) impinging on the array [3-2]. As in [3-1] we model this effect by multiplying, on an element-by-element basis, the signal covariance matrix $\mathbf{R}_s$ and the interference matrix $\mathbf{R}_i$ by the matrix $\mathbf{R}_{\text{coh}} = \text{toeplitz}(\mathbf{r}_{\text{coh}})$ where the vector $\mathbf{r}_{\text{coh}}$ contains samples from the spatial coherence function. In our case we choose

$$r_{\text{coh}}[n] = \exp(-(n-1)/N_{\text{coh}})$$

for simplicity. The first entry ($n=1$) is unity. N_{coh} is defined as the coherence length, measured in sensor samples. For $N_{\text{coh}} \rightarrow \infty$ we have the usual assumption of perfect coherence. For $N_{\text{coh}} \rightarrow 0$ we have zero field correlation between adjacent sensors. We are interested here in modest coherence lengths of the order of the length of the array, $N_{\text{coh}} \sim N$.

Because the NB signal and interference contributions are no longer, in linear algebra terms, rank-1, it may be envisaged that the impact of coherence effects on the MVDR processor can be quite complex. However, after some considerable period of trial-and-error it was deduced that provided the SNR was sufficiently low, the STMV-SA1 beamformer performed roughly the same as the CB *in the same environment*, though of course both beamformers have inferior performance relative to processing fully coherent wavefronts. For a close interferer the SA1 beamformer retains some gain against the interference, but some loss in output SNR (recall the signal and interference are assumed to be well-separated in frequency).

Fig. 3.12 shows the AGI(SA1, CB) curves corresponding to beamforming at the signal frequency for the same SA1 beamformer as above ($N=24$, $N_s=21$, $N_a=4$) operating with SNR = 10dB and INR from -10 to +30dB. The coherence length is set at $N_{\text{coh}} = N$ 24. This is quite a short coherence length. In Fig. 3.12 we see that for $v_1 > 0.5$ the SA1 performance is a dB or so better than the Hamming-shaded CB. At lower interference spatial frequencies we have a similar effect to that seen earlier.

In Fig. 3.13 we have the corresponding set of curves, INI(SA1,CB) for beamforming at the interference frequency. We note modest gains at low spatial frequencies dropping to a dB or so at higher values of v_1 . Overall these two figures demonstrate that the SA1 processor is not unduly compromised by coherence effects.

If the SNR is increased to, say, 20dB or higher, the curves (not shown) become decidedly more complex with a general trend to inferior performance by the SA1 beamformer. However, if we increase the coherence length to $N_{\text{coh}}=100$ then these type of problems do not arise until the SNR is at least 30dB. It would seem then that coherence effects do not compromise the SA1 beamformer (relative to the CB in the same environment) if the SNR is sufficiently low for a given coherence length and the limiting value of SNR increases with coherence length. Though not covered in any detail here, it

was observed that the impact of the coherence effects could be reduced somewhat by lowering the number of adaptive DOF in the SA1 processor.

3.6 Conclusion

It has been demonstrated that the STMV-SA1 technique applied to an array with (known) missing sensors is susceptible to problems caused by plane wave interference from directions corresponding to nulls in the subarray beampatterns. However, a slight modification of the subarray structure alleviates this problem and leads to the SAIM beamformer. The partially-adaptive SAIM applied to an array with missing sensors is demonstrated to have a performance almost matching that of the fully-adaptive version with no missing sensors. With strong interferers and missing sensors, its performance is superior to the Hamming-shaded CB. This is perhaps one area where it is clear that the adaptive processing is superior to non-adaptive.

On the other hand, the presence of unknown sensor perturbations and a strong signal from the desired direction can lead to inferior SA1 performance. There is both a signal cancellation effect and an associated increase in the weight norm which increases the output noise level at all frequencies. In the presence of interference, the overall performance, in comparison to the CB, depends on the relative magnitudes of the signal-related and interference-related effects. If we assume the signals of most interest are weak, then the STMV-SAIM beamformer is the beamformer of choice.

In the matter of the impact of the coherence length of the impinging wavefronts, it has been demonstrated that the SA1 processor remains competitive with the Hamming-shaded CB in the same environment provided the SNR of the desired signal is sufficiently low. In our experiments with a coherence length roughly the same length as the array, this limiting input SNR was shown to be 10 to 20dB (20 to 30dB "in the beam"). The threshold level increases with coherence length. Beyond the threshold value the situation is quite complex and cannot easily be summarised.

3.7 Appendix: Mathematical Details

The SA1M beamformer differs from the SA1 in (a) the choice of the $\mathbf{K}$ matrix and (b) the effective steering vector in the SA-MVDR process.

Following (2.11) we may write an expression for the N element weight vector as

$$w_{sa} = \frac{\mathbf{K} (\mathbf{K}^H \mathbf{R} \mathbf{K})^{-1} \mathbf{K}^H \bar{\mathbf{l}}_{N_s}}{\bar{\mathbf{l}}_{N_s}^H \mathbf{K} (\mathbf{K}^H \mathbf{R} \mathbf{K})^{-1} \mathbf{K}^H \bar{\mathbf{l}}_{N_s}} \quad (3.A1)$$

The $N \times N_a$ matrix $\mathbf{K}$ is general. In the STMV-SA1 beamformer the columns contain overlapped sub-vectors $\bar{\mathbf{l}}_{N_s}$. In the STMV-SA1M method of this chapter an additional unity entry is made immediately above the $\bar{\mathbf{l}}_{N_s}$ sub-vector in the rightmost column, i.e. $\mathbf{K}[N-N_s, N_a] = 1$. This choice is arbitrary. The effective "steering vector" is seen as $\mathbf{K}^H \bar{\mathbf{l}}_{N_s}$ which in the case of the STMV-SA1 processor is directly proportional to $\bar{\mathbf{l}}_{N_s}$. The complicated-looking denominator is simply a scaling factor to ensure unity signal gain. In order to deal with missing sensor(s), the appropriate row(s) of $\mathbf{K}$ is(are) simply replaced by zeros.

We remark in passing that the SA structure in this report has $\mathbf{K}$ containing zeros and ones only. It may be of interest to ask the question of whether there is an "optimum" distribution of 0's and 1's in a $\mathbf{K}$ matrix of a given size.

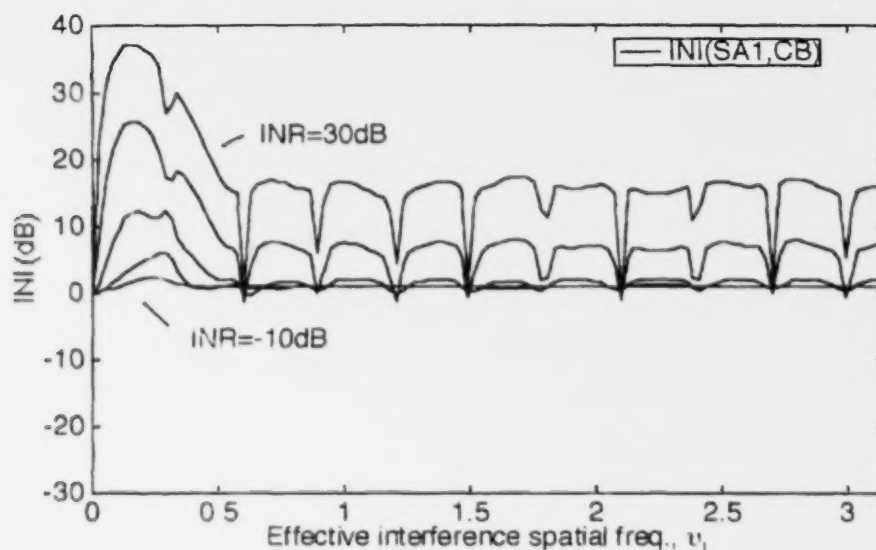


Fig. 3.1 Missing sensor effect. INI(SA1, CB). Sensor #7 missing. $N_s=21$, $N_d=4$, SNR=-10dB. INR={-10 to +30dB} Note problematic deep nulls at $2\pi n/N_s$. Note gain against CB.

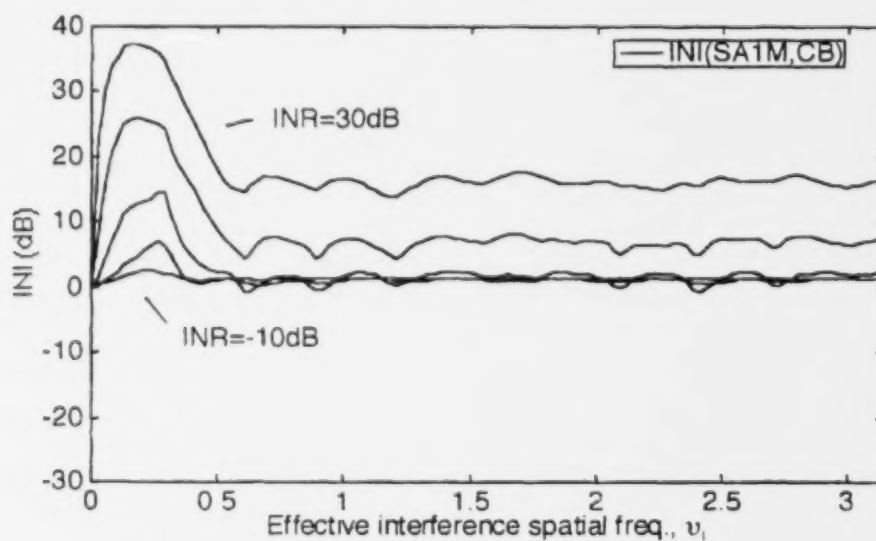


Fig. 3.2 As above with modified K matrix. (See text for details). Troublesome nulls have largely disappeared.

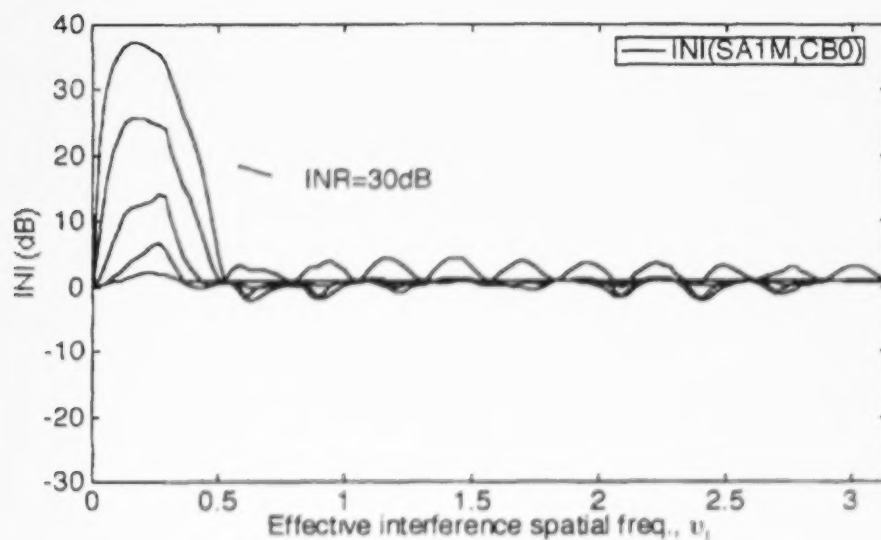


Fig. 3.3 As Fig. 3.2 but comparison with unperturbed array CB0. Only slight effects of missing sensor are visible,

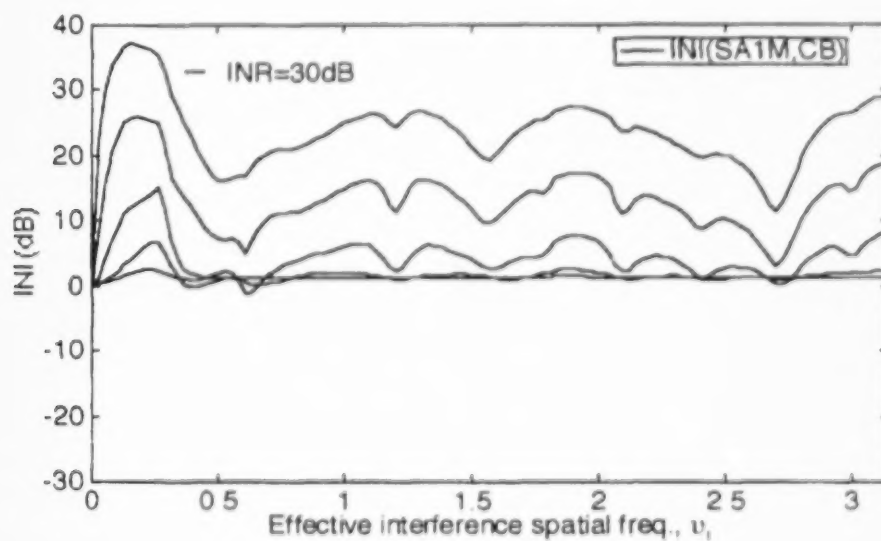


Fig. 3.4 Multiple missing sensors . Sensors {7,11,17}, missing. Similar results to Fig. 3.2.

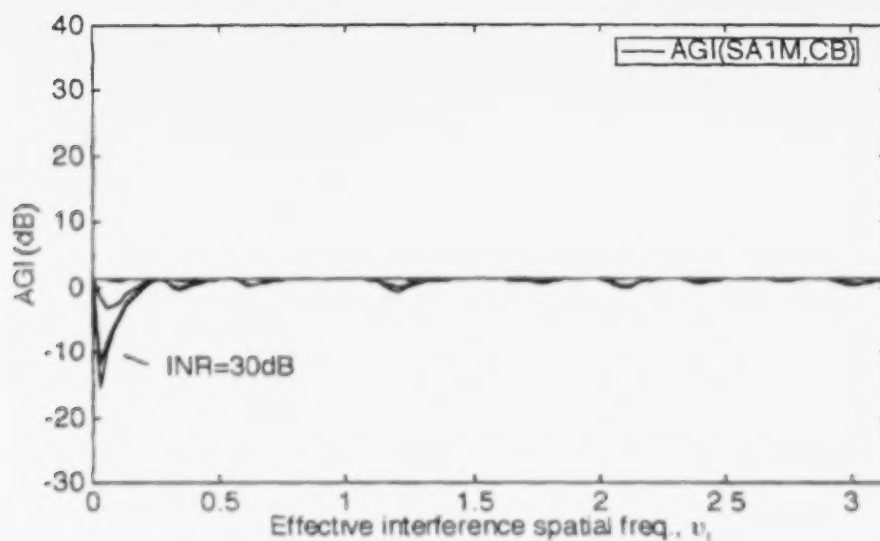


Fig. 3.5. AGI(SA1M, CB). Three missing sensors. Little impact of INR or missing sensors.

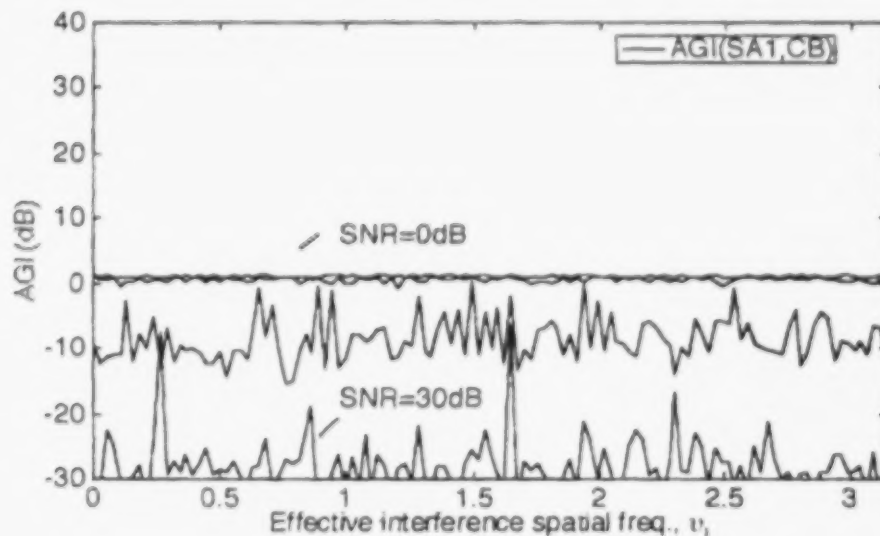


Fig. 3.6. Sensor perturbation effects. AGI(SA1, CB). DVR = -20dB. INR = -20dB. Note impact of increasing SNR.

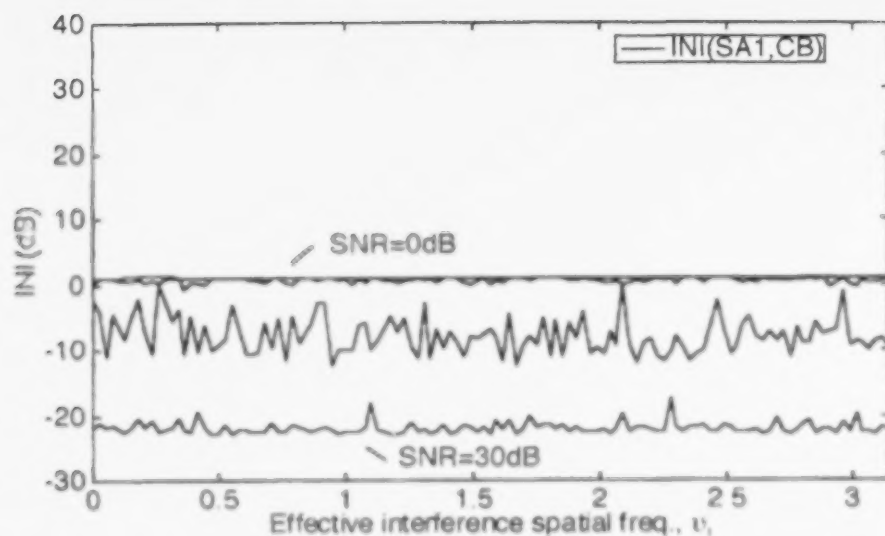


Fig. 3.7 As Fig. 3.6 but INI(SA1, CB) at interference frequency. INR=-20dB
Note impact of SNR.

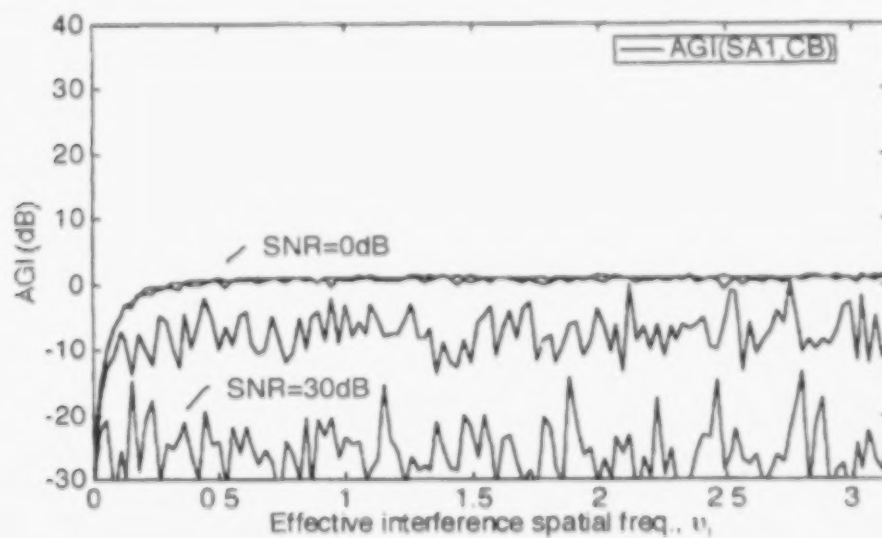


Fig. 3.8. As Fig. 3.6 but in presence of 30dB interferer. Little change except at low spatial frequencies.

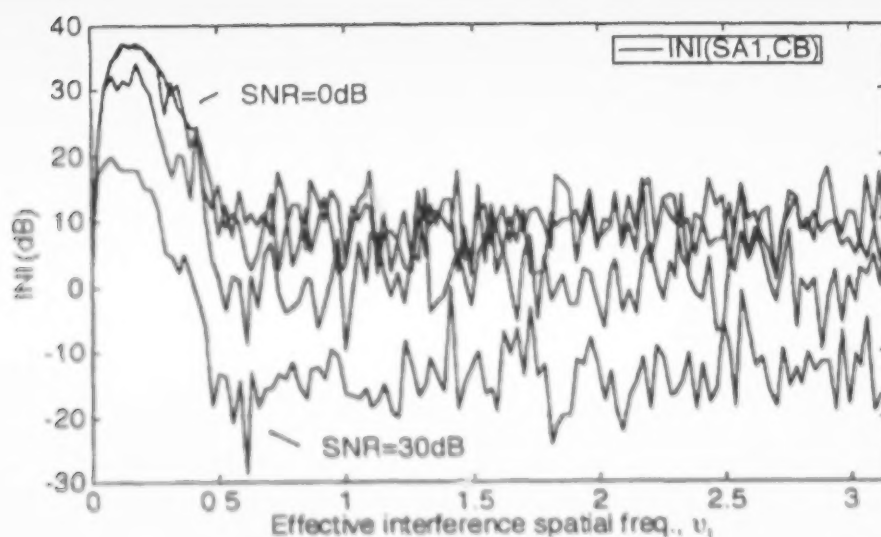


Fig. 3.9 As Fig. 3.7 but with INR=30dB. Improvement of SA1 at low SNR. Deterioration at high SNR.

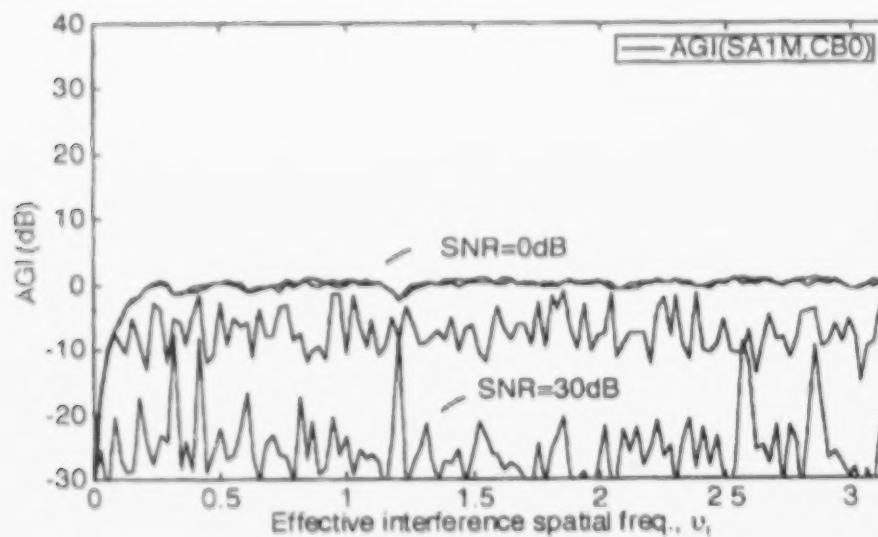


Fig. 3.10 Both sensor perturbations (DVR = -20dB) and missing sensors{7,11,17}. INR = 30 dB. Note comparison is with ideal array, CB0.

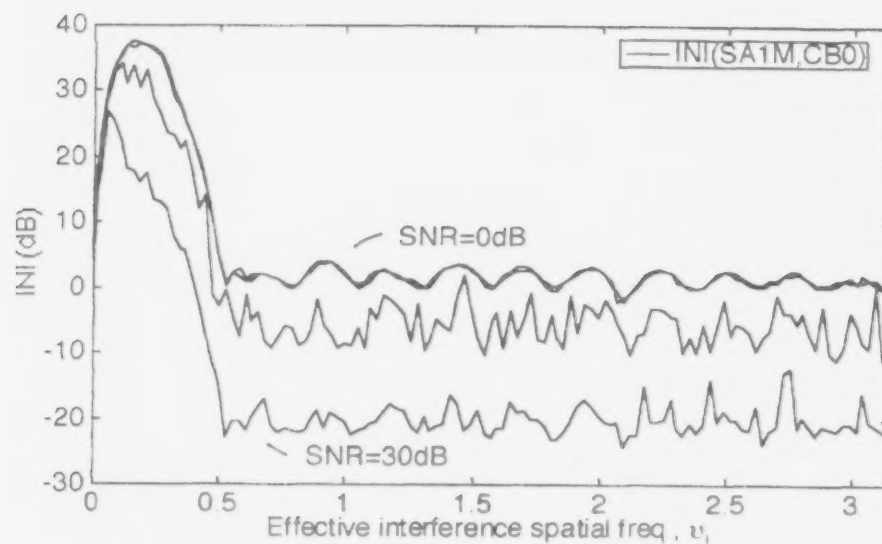


Fig. 3.11 As Fig. 3.10 but at interference frequency. Note comparison is again with ideal array, CB0.

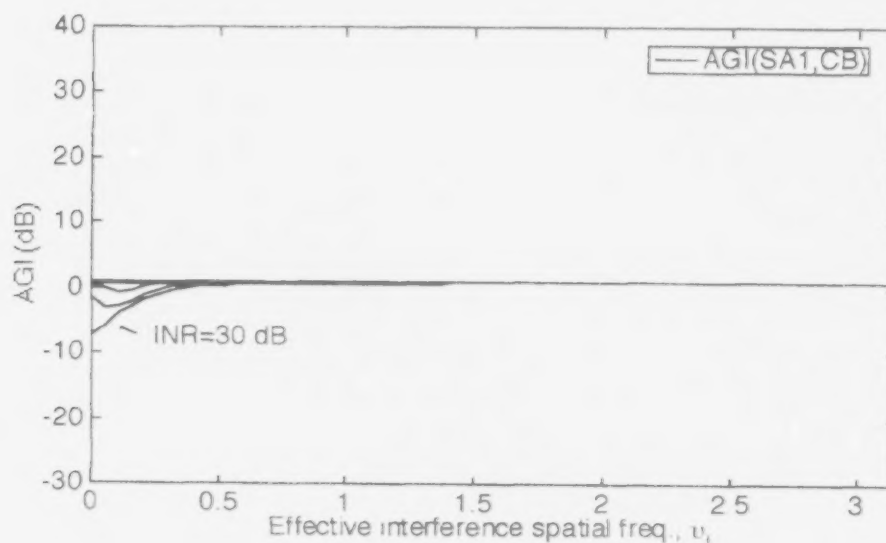


Fig. 3.12 Coherence length effect. $N_{coh} = 24$, $N = 24$, $N_s = 21$, $N_d = 4$. $SNR = 10$ dB $INR = \{-10, 0, 10, 20, 30\}$ dB. Little impact of coherence effects.

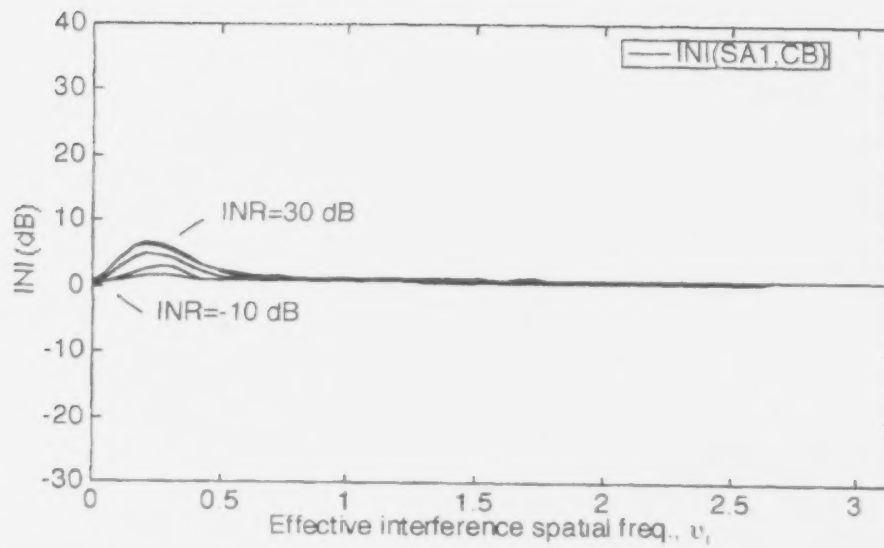


Fig. 3.13 As Fig. 3.12 but at interference frequency. Some improvement in gain at low spatial frequencies.

4. FINITE-DATA EFFECTS: THE MISADJUSTMENT PHENOMENON IN STMV-SA BEAMFORMING

4.1 Introduction

In this chapter we briefly investigate the misadjustment effect in STMV-SA beamforming caused by the use of a finite number of Fourier snapshot vectors in forming the (WB) STCM matrix estimate from a single time-series observation. In the NBMV case, there is a similar limitation on the number of vectors imposed by the non-stationarity of the field which restricts the overall observation time and hence the number of time-series snapshots available, each of which provides only one snapshot vector to the (NB) covariance matrix estimate.

We shall demonstrate a somewhat surprising effect: For a narrowband signal, the beamformer output is more stable in the STCM formulation (and its subarray STMV-SA variants) than in the corresponding NBMV case. This is surprising in the fact that only a single Fourier vector in the STCM bears signal energy, whereas all of them do in the NBMV case. Thus the latter might be expected to give "better" weight vector estimates. We shall also demonstrate that the STMV beamformer is less susceptible to the signal-cancellation effect in the presence of correlated interference, though its performance is still seriously compromised at higher SNR values.

We shall also briefly touch on the use of covariance matrix estimates based on exponential averaging.

4.2 Problem formulation

In the basic STMV case we form an $N \times N$ STCM matrix estimate, $\hat{\mathbf{R}}$, from a single time-series observation according to equation (2.1). It is based on a $N \times M$ data matrix, $\mathbf{Y}$, each column of which corresponds to a Fourier (i.e. temporal frequency) vector, one from each of M FFT bins covering the processing band of interest. We again assume we have a tonal signal in our chosen steering direction, at frequency, f_0 , of power P_s "in the bin". It contributes only to a single Fourier vector. The other $M-1$ vectors represent random self-noise contributions with mean power, P_n .

For the benchmark NBMV beamformer we form a covariance matrix, commonly called a cross-spectral-density matrix (CSDM), estimate, $\hat{\mathbf{R}}_{NB}$, with

$$\hat{\mathbf{R}}_{NB} = \mathbf{Y}_{NB} \mathbf{Y}_{NB}^H \quad (4.1)$$

where $\mathbf{Y}_{NB}$ is again a $N \times M$ matrix but whose columns are now M independent realisations of the Fourier coefficient vector at the signal frequency, f_0 , again pre-steered in the steering direction. This implies M time-series observations with a concomitant reduction in the ability to track highly dynamic fields.

Based on these covariance matrix estimates we can perform partially adaptive beamforming using weights computed for the STMV-SA method, for example, by

$$\hat{\mathbf{w}}_{SA} = \frac{\mathbf{K}(\mathbf{K}^T \hat{\mathbf{R}} \mathbf{K})^{-1} \bar{\mathbf{I}}_{Na}}{\bar{\mathbf{I}}_N^T (\mathbf{K}^T \hat{\mathbf{R}} \mathbf{K})^{-1} \bar{\mathbf{I}}_{Na}} \quad (4.2)$$

Equation (4.2) is simply (2.11) repeated here for convenience. $\mathbf{K}$ is the $N \times N_a$ selection matrix detailed earlier in the discussion surrounding (2.4). There is the obvious weight vector, $\mathbf{w}_{NB}$, for the NBMV-SA case corresponding to (4.2) with $\mathbf{R}$ replaced by $\mathbf{R}_{NB}$. In both cases, the number of adaptive degrees of freedom is $N_a - 1$.

Using these weights we can estimate the signal component for the STMV-SA beamformer via,

$$\hat{s} = \hat{\mathbf{w}}_{SA}^H \mathbf{y}_0 \quad (4.3)$$

where $\mathbf{y}_0$ is understood to represent the single signal-bearing Fourier vector in the (WB) STCM formulation. Again there is the corresponding output estimate for the NBMV beamformer where $\mathbf{y}_0$ is now understood to be the "most recent" vector involved in the CSDM estimate.

4.3 Numerical Results

4.3.1 Signal plus noise

We start examining the beamformer output estimate, $\hat{s}$, for the two partially adaptive, subarray (SA) beamformers discussed above. The (pre-steered) signal input vector is set to be $s\bar{1}$ where $s = 10 + j0$. The uncorrelated gaussian self-noise power ("in the bin") is $P_n = 1$, so $\text{SNR}_{\text{in}} = 20\text{dB}$. There is no interference. We use $M=32$ Fourier vectors in forming the STCM and CSDM estimates.

Initially, we arbitrarily choose $N = 16$ sensors, subarray length of $N_s=1$ with offset $=1$ giving $N_d = N = 16$ subarrays. This SA arrangement is actually recognised as classic, fully-adaptive, STMV and NBMV beamforming. For comparison purposes we also include a Hamming-shaded conventional beamformer, CB.

In Fig. 4.1 we see the real and imaginary parts of $\hat{s}$ for 100 independent realisations of the beamforming process. It is immediately obvious that the CB beamformer is the best performer. It alone has the estimate of $\hat{s}$ centred on the desired value of $10+j0$. The MVDR-based outputs exhibit a downward bias effect. This bias is explored in [4-1] and we have for both SA methods

$$E(\hat{s}) = (1 - \frac{N_s-1}{M}) s \quad (4.4)$$

which in our case yields $E(\hat{s}) = 5.3+j0$ which agrees with the MVDR results in Fig. 4.1.

Equally obvious from Fig. 4.1 is the fact that the NBMV results are much noisier than the other two. This may be counter-intuitive based on the notion that the signal component is present in each of the $M=32$ Fourier coefficient vectors, and the weight estimates might be expected to be more stable than the STMV method. This is just not the case. We can again use [4-1] as a basis for exploring the variance issue more carefully as follows:

For the NBMV-SA case we may write

$$\text{var}(\hat{s}) = (1 - \frac{N_s-1}{M}) (P_{s0}^* + \frac{N_s-1}{M} P_{s0}) \quad (4.5a)$$

and for the STMV-SA case we may write

$$\text{var}(\hat{s}) = \left(1 - \frac{N_a - 1}{M}\right) \left(P_{no}^* + \frac{N_a - 1}{M^2} P_{si}^*\right). \quad (4.5b)$$

The $()^*$ notation implies a mean output power given by an ideal MVDR beamformer based on the underlying true (rather than estimated) covariance matrix. An extra factor of M is inserted in the denominator of the P_{no}^* term in (4.5b) due to the fact that the signal power appears in only $1/M^{\text{th}}$ of the STCM snapshots. This has a significant impact. We should also emphasise that the above are appropriate for all SA cases with any choice of M and N_a and for all interference and noise distributions (with P_{no}^* understood to include both noise and interference effects).

Again following [4-1] we may define an output signal to noise ratio for our SA beamformers as

$$SNR_{out} = (E(\hat{s}))^2 / \text{var}(\hat{s}). \quad (4.6)$$

For the NBMV-SA we have explicitly from the above

$$SNR_{out} = \frac{(1 - \frac{N_a - 1}{M}) SNR_{out}^*}{(1 + \frac{N_a - 1}{M} SNR_{out}^*)} \quad (4.7a)$$

and for the STMV-SA we have

$$SNR_{out} = \frac{(1 - \frac{N_a - 1}{M}) SNR_{out}^*}{(1 + \frac{N_a - 1}{M^2} SNR_{out}^*)}. \quad (4.7b)$$

where $SNR_{out}^* = P_{so}^* / P_{no}^*$ is the output SNR for the underlying ideal beamformer.

In Fig. 4.2 we plot the output SNR's estimated via (4.6) from 200 realisations of the process of the type shown in Fig. 4.1 for various input signal levels, s . We also plot the theoretical output SNR's based on (4.7) together with the corresponding CB results. In this application we know that $SNR_{out}^* = NP_s / P_n = N|s|^2 / P_n = N SNR_{in}$. SNR_{in} is varied in 5dB steps from -10 dB to +20 dB. As in Fig. 4.1, we use the fully-adaptive SA beamformers ($N_a = N$), otherwise known as the NBMV and STMV beamformers.

In Fig. 4.2 we observe immediately that the CB results are superior with a true linear relationship (in dB terms). For the STMV method, at higher values of SNR_{in} , SNR_{out} tends to plateau at (from (4.7b)) $(1 - (N_a - 1) / M) M^2 / (N_a - 1)$ or about 15dB, independent of SNR_{in} . The NBMV results plateau some 15dB lower; its SNR_{out} value can never exceed about 0dB, irrespective of input SNR. This is very low. We should also note again the close agreement between the theoretical and experimental results in Fig. 4.2.

We now implement partially-adaptive SA beamforming with $N_a = 4$ subarrays and $N_s = 14$ sensors per subarray. This will be recognised as an SA1-type beamformer from the previous chapter. The SA2 beamformer, with non-overlapping subarrays, was found therein to be inferior and will not be considered here. In Fig. 4.3 we plot SNR curves corresponding to Fig. 4.2 and we see a general improvement in SNR_{out} for the SA beamformers, particularly at higher input SNR's. This is due entirely to the reduction in the value of N_a . Again we should also note the close agreement between the theoretical and experimental results.

In passing, we may look at the impact of applying the STMV-SA weight vector at a frequency other than the signal frequency, f_0 . The chosen Fourier vector is now "noise-only" but the weight vector is still influenced by the presence of a strong signal component in the STCM. This in turn results in an increase in the noise-power output as predicted by (4.5b). In Fig. 4.4 we show noise output power estimates as a function of SNR_{in} . The scenario is exactly as in Fig. 4.3. For the NBMV-SA curve, the noise data may be interpreted as the noise output at the signal frequency. Note the agreement between the experimental and theoretical results. The STMV-SA results show in this scenario that the STMV-SA beamformer is superior even to the CB for SNR_{in} below about 10 dB.

4.3.2 Signal plus uncorrelated interference plus noise

Adding a strong interferer ($\text{INR} = 20$ dB) at some different temporal frequency with an effective spatial frequency of $7\pi/16$ (well outside the mainlobe of the CB), yields the results of Fig. 4.5. There is no apparent impact of the interference on the STMV-SA or CB curves. (We would not expect any difference in the NBMV-SA results as in this scenario the CSDM will not contain any interference contributions). This demonstrates

that the STMV-SA misadjustment effect is only weakly related to interference contributions not co-located at the signal frequency.

4.3.3 Multipath effects. Signal plus noise plus correlated interference.

Finally we investigate the impact of the multipath phenomenon where we have an interferer at the signal frequency and correlated with it. (We recall from the discussion in the previous chapter that in the STCM-based method any interference at the signal frequency will appear to be correlated). In Fig. 4.6(a) we rerun the previous experiment but have a 20 dB INR fully-correlated interferer situated at an effective spatial frequency of $4.5\pi/13$.

In Fig. 4.6(a) we see the severe consequences of the multipath effect which causes signal cancellation. At higher input SNR's, the STMV-SA output SNR has dropped by about 10dB while the NBMV-SA output SNR has dropped by about 50 dB. These are really significant effects, though it is true to say that the STMV-SA method is more robust than its NBMV-SA counterpart, primarily due to the presence of a higher effective white noise level in the underlying covariance matrix (see equation (2.10)) which masks the interference effect to some degree. Note that the signal cancellation effect is most severe at higher SNR's and tends to dwarf any misadjustment effect.

Moving the correlated interference slightly such that its effective spatial frequency is reduced to $4\pi/13$ yields the results of Fig. 4.6(b) which are identical to the non-multipath results of Fig. 4.3. Following our discussion in Chapter 2, this is to be expected as the interference is now situated at one of the nulls of the SA beampattern (recall $N_s = 13$) and thus the interference is entirely eliminated from the ensuing MVDR beamforming process.

4.4 Comment on exponential averaging.

In the foregoing we have compared STCM-based beamforming vs. CSDM-based beamforming where both matrices are estimated from a block of M Fourier vectors. However, it is also possible to form a CSDM estimate via an exponential averaging operation as follows:

$$\hat{\mathbf{R}}'_{NB} = \alpha \hat{\mathbf{R}}'^{t-1}_{NB} + (1 - \alpha) \mathbf{y}_t \mathbf{y}_t^H. \quad (4.8)$$

In this expression, $\hat{\mathbf{R}}'_{NB}$, is the current overall estimate of the matrix at time, t , with $\mathbf{y}_t$ being the "current" Fourier vector. In the "blockwise" (or "sample matrix") CSDM version of (4.1) we effectively retain a vector with no attenuation until it is of "age" M when it is totally forgotten.

It is of interest to be able to link the forgetting factor α of (4.8) to the blocksize, M , for CSDM-based beamforming. During the experimental work for this section, some simulations were run using various values of α and various values of M for CSDM-based beamforming with the goal of identifying (roughly) how α and M relate for a given output SNR (i.e. misadjustment) performance. Very roughly it was found that equivalent SNR performance is obtained if [4-2]

$$M \approx \frac{2}{1 - \alpha}. \quad (4.9)$$

The use of exponential averaging is also possible in the STCM mode where the overall STCM is the weighted sum of previous STCM's using a forgetting factor along the lines of (4.8). In this work, in the spirit of quasi-instantaneous convergence, we have simply used the current STCM matrix estimate only (forgetting factor = 0).

4.5 WB signals.

The above has concentrated on NB, tonal signals. The case of WB signals is implicit in the results of [4-1]. All other things being equal, we expect the misadjustment effect to be worse in the WB case as the signal component appears in all columns of the data matrix used in forming the STCM.

4.6 Summary

This section has focussed on the misadjustment effect, caused by finite data availability, as it impacts the output SNR performance for a tonal signal. For uncorrelated tonal interference, we have shown that the misadjustment effect is a strong function of the

input SNR and is more serious at higher values. It is reduced as the number of adaptive degrees of freedom ($N_a - 1$) is reduced. It is a very weak function of interference strength (assuming, of course, the interference is not "too close" to the steering direction).

We have also demonstrated that the analytical results of Krolik and Swingler [4-1], based originally on a CSDM assumption, are easily modified for the STMV-SA case. Most importantly we have shown that the STMV-SA is significantly more robust against misadjustment effects than the corresponding NBMV-SA case, even if the same number of snapshot vectors, M , are used in each case. For high input SNR's, the output SNR for the STMV-SA is better by a factor of about M .

If correlated interference occurs at the signal frequency (or is simply co-located in frequency in the STMV-SA) case, both MVDR beamformers suffer a significant loss in SNR. The STMV-SA beamformer is again more robust than the corresponding NBMV beamformer though its performance can be seriously compromised, especially at higher SNR values.

A few comments have been made on the equivalence of beamforming based on sample CSDM estimates versus exponentially-averaged CSDM matrices.

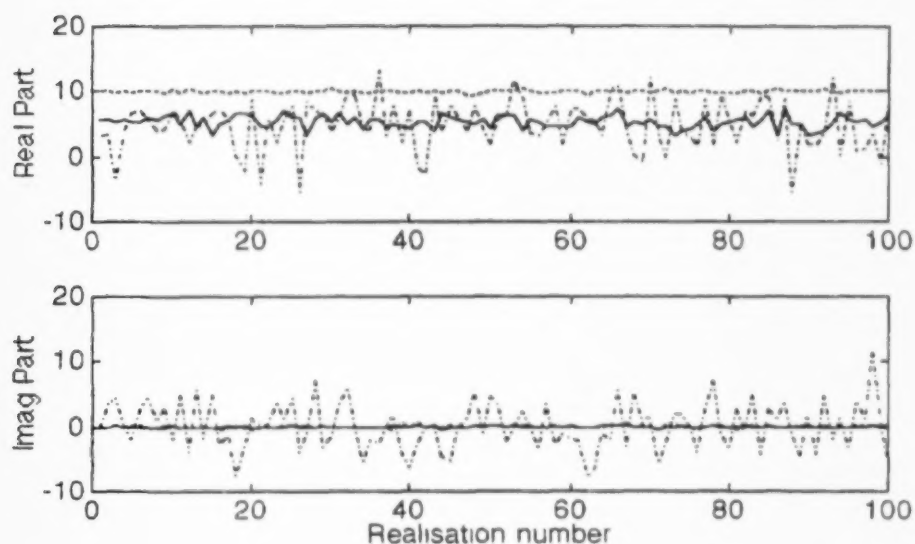


Fig. 4.1 Beamformer outputs. STMV = __, NBMV = -.-, CB = -.-

$P_s=100$, $P_n=1$, $N=N_d=16$, $M=32$. Note the bias in the MV results and the noisiness of the NBMV.

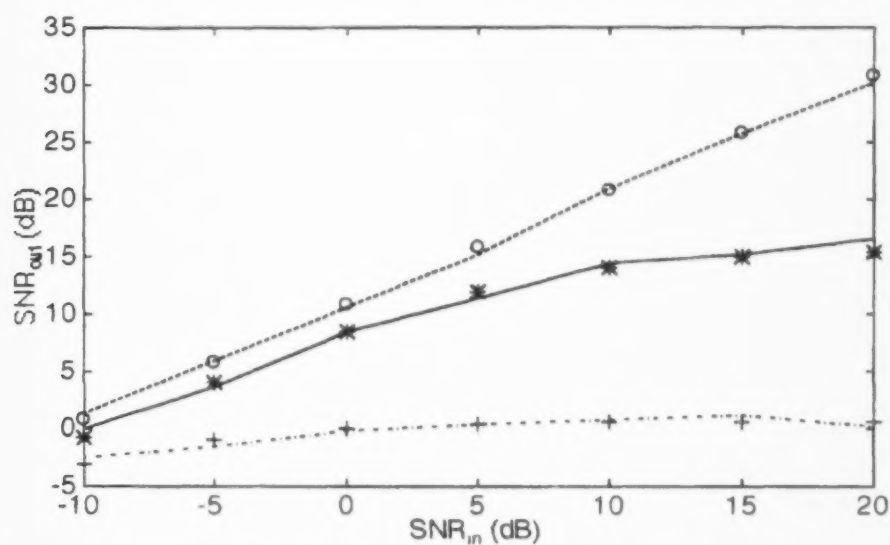


Fig. 4.2 SNR curves for fully adaptive ($N_d=N$) beamformers. STMV (___), NBMV (-.-), CB (-.-). Theory values indicated by markers. Based on 200 realisations.

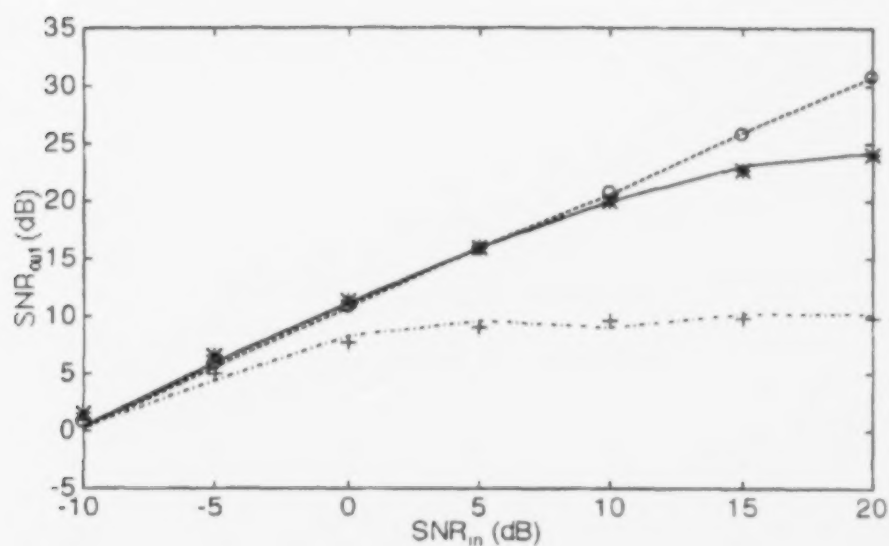


Fig. 4.3 SA processing. As Fig. 4.2 but with $N_d=4$, $N_c=13$. Note general improvement in output SNR for SA techniques. STMV-SA (—), NBMV-SA (---), CB (· · ·)

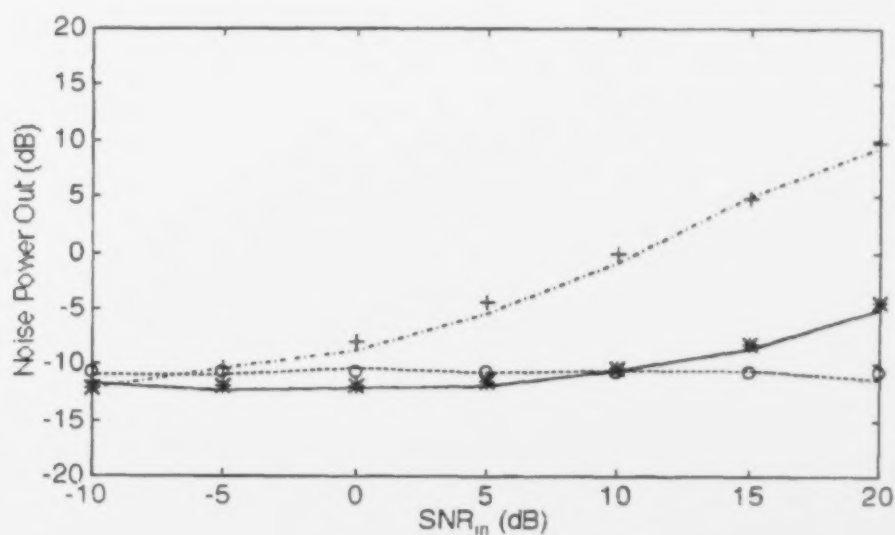


Fig. 4.4 Noise only output powers. SA methods are affected by SNR_{in}. STMV-SA (—), NBMV-SA (---), CB (· · ·)

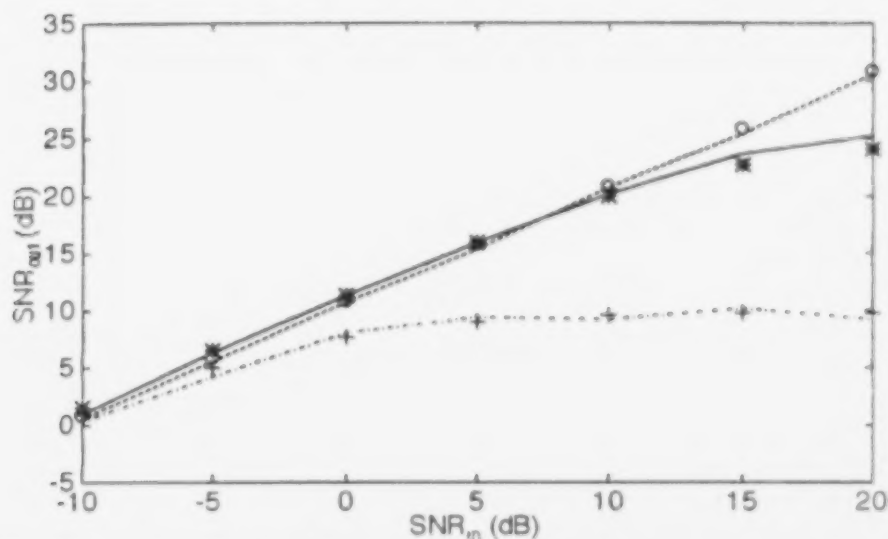


Fig. 4.5 As above but with interferer present (INR=20dB, spatial frequency $7\pi/16$). No apparent effect. STMV-SA (—), NBMV-SA (---), CB (---)

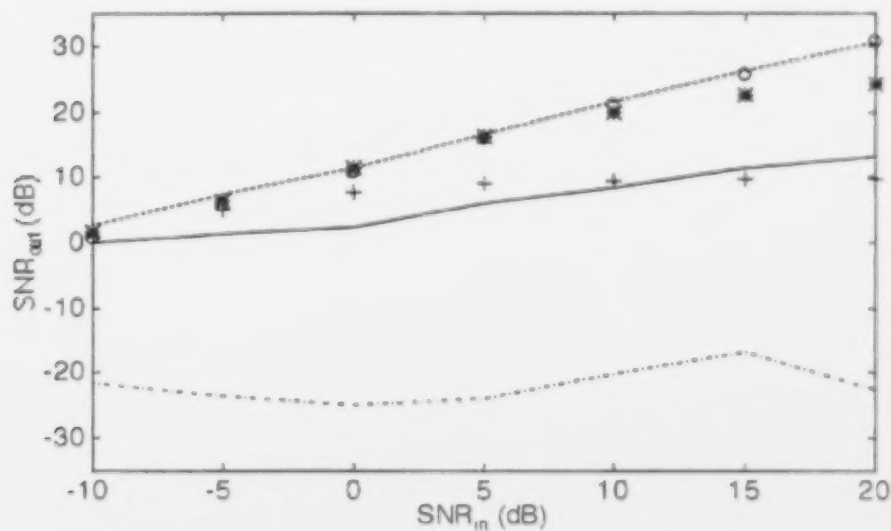


Fig. 4.6(a) Multipath effect. INR=20dB at $4.5\pi/13$. $N=16$, $N_a=4$, $N_s=13$. Note catastrophic signal cancellation effect for NBMV. Markers deliberately left at their non-multipath levels for reference. STMV-SA (—), NBMV-SA (---), CB (---).

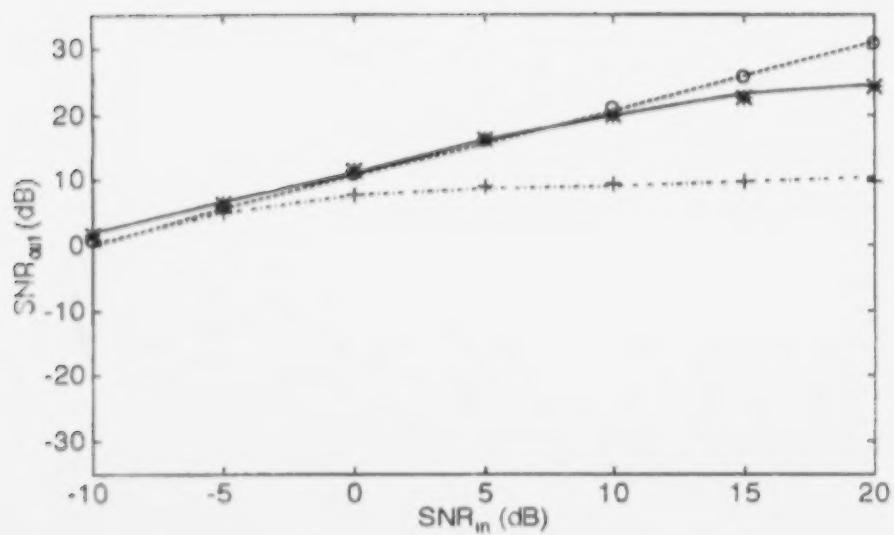


Fig. 4.6(b) As Fig. 4.6(a) but with interferer moved slightly to $4.0\pi/13$. Multipath signal cancellation effect disappears. STMV-SA (—), NBMV-SA (---), CB (- -)

5. THE DOMINANT-MODE REJECTION BEAMFORMER

5.1 Introduction

We now turn our attention to the adaptive Dominant-Mode Rejection Beamformer (DMRJ) originally suggested by Abraham and Owsley [5-1] and examined more thoroughly very recently by Messerschmidt and Gramann [5-2]. The DRMJ method was proposed as a NB method which could be usefully employed with reduced observation times. This approach is attractive in terms of our work which is based on the notion of "instantaneous convergence" of an adaptive beamformer (ABF), i.e. applicable to a single time-series snapshot.

We will demonstrate that the DMRJ process is not well suited to the WB steered-covariance approach which underlies all of the procedures examined in this report. Indeed, it is true to say that using the DMRJ approach in conjunction with the WB steered covariance matrix estimates obtained from a single time-series observation can lead to catastrophic results. Even in the simplest case of a single NB (tonal) signal in white noise, the DMRJ-BF performs significant cancellation of the desired signal while simultaneously increasing the output levels of the background noise. In short, for the application under study here, the DMRJ-BF is not competitive with other steered covariance method(s) based on a single time-series (finite data) observation.

5.2 DMRJ Basics

The premise of the DMRJ beamformer is quite simple. Given a covariance matrix estimate, $\hat{R}$, we retain only the dominant modes (i.e. that part of the matrix related to dominant signals) by performing a SVD decomposition on $\hat{R}$. We form a dominant mode matrix approximation, $\hat{R}_D$, by retaining only the D largest singular values. The other modes are replaced by the equivalent of a white noise background by adding an appropriately weighted identity matrix.

In more mathematical terms, we perform a SVD decomposition on the $N \times N$ Hermitian covariance matrix estimate $\hat{R}$:

$$\hat{R} = U S U^H \quad (5.1)$$

where $\mathbf{U}$ is $N \times N$ and contains the N "singular vectors" and $\mathbf{S}$ is a diagonal matrix whose entries are the ordered set of singular values. The D largest singular values are then used to form the overall dominant mode approximation:

$$\hat{\mathbf{R}}_D = \sum_{i=1}^D s_i \mathbf{u}_i \mathbf{u}_i^H + \epsilon \mathbf{I}_N \quad (5.2)$$

where $\mathbf{u}_i$ is the i^{th} column of $\mathbf{U}$ (the i^{th} singular vector) and s_i is the i^{th} largest singular value (the ii^{th} element in $\mathbf{S}$). The weighting factor ϵ is "appropriately chosen", (one of the subtleties of the DMRJ method); here we simply let ϵ be equal to the smallest singular value, s_N , which can be taken as a measure of the overall background noise level. The specific choice of ϵ is not felt to be particularly important in this work.

The choice of D is dependent on the knowledge (guess?) of the number of dominant signals in the field. Again, however, its specific choice is not too important. Having chosen D and ϵ , the matrix $\hat{\mathbf{R}}_D$ replaces $\hat{\mathbf{R}}$ in all subsequent MVDR operations leading to the calculation of the adaptive weights. These are then used in conjunction with the original data to yield beam outputs in the usual manner.

5.3 Application to the WB steered covariance method (ST-DMRJ).

For our purposes, we simply let $\hat{\mathbf{R}}$ be the pre-steered WB covariance matrix described in detail in earlier chapters. We then form the dominant mode version of it and use that to find the adaptive weight vector which is itself then used to compute the output associated with each of the narrowband (Fourier) vectors in the original data, again as described earlier.

5.4 A Simple Finite-data Example

As a simple example with finite data, the field might contain only a single strong 375 Hz tonal at bearing $\theta_0 = -20^\circ$ in uncorrelated noise, with an expected output SNR from an unshaded CB beamformer of 30dB, "in the bin". The array has $N = 12$ sensors with an inter-element spacing of 1m. The propagation speed is 1500m/s. A single time-series observation sampled at 1kHz contains 128 data points and the number of Fourier

(FFT) bins across the chosen processing band of 280-400Hz is thus sixteen. As there is only a single source we would select $D=1$ for the DMRJ beamformer.

Fig. 5.1 shows a family of SONARGRAMS representing output NB beampowers from each of 12 beams (equi-spaced in $\sin(\text{bearing})$) for 24 consecutive snapshots. In each subplot the horizontal axis is frequency, the vertical axis is time (measured in snapshots). We immediately see that in the beam at -19.47° , the one nearest the true signal direction, there is a significant and more or less uniform increase in beampower¹ across all the frequencies in the band. In this beam there is no discernible "peak" in observed output power at 375Hz. This is a disconcerting effect as we know that the output SNR should be about 30dB at this frequency.² For comparison purposes, Fig. 5.2 shows the same data set processed with the STMV-SAI beamformer. Here we see a strong line emerge at 375Hz in the beam at -19.47° , though it too exhibits a marked increase in background noise output in that beam.

This example quite clearly demonstrates that the DMRJ method is incompatible with the use of the STCM matrix estimated with finite-data from a single time-series snapshot. The explanation is that in the signal direction the DMRJ-MVDR process becomes geared to eliminating the (large) energy associated with the sole singular vector retained in the $\hat{\mathbf{R}}_D$ matrix. This yields a unity-gain weight vector with a large norm. Hence we have simultaneously (relative) signal cancellation and an elevation in the background noise level which together yield the large and relatively uniform power outputs observed across all frequencies in the beam at -19.47° on Fig. 5.1. Recall that the same weights for a given snapshot and a given beam are used across all frequencies.

It should be stated that it was observed that this noise-elevation/signal-cancellation problem diminished as the number of FFT bins increased (effectively increasing either the BW and/or the observation time). However, it was felt that the ST-DMRJ results of Fig. 5.1 were of such low quality as to rule out any further interest in that method for our application.

¹Note beampower colorbar at extreme bottom right of figure is in dB.

²This effect was not ameliorated on changing the source bearing to exactly -19.47 degrees

5.5 Summary

We have demonstrated that the dominant-mode rejection beamformer is not competitive with other adaptive beamformers in the type of application under study here. Unlike these other methods, all based on the wide-band steered-covariance matrix obtained from a single time-series snapshot, the ST-DMRJ beamformer can exhibit severe noise-elevation/signal-cancellation effects for even the simplest case of a single tonal signal in white noise.

Overall, it is recommended that the ST-DMRJ beamformer be eliminated from further study in connection with STCM-based adaptive beamforming

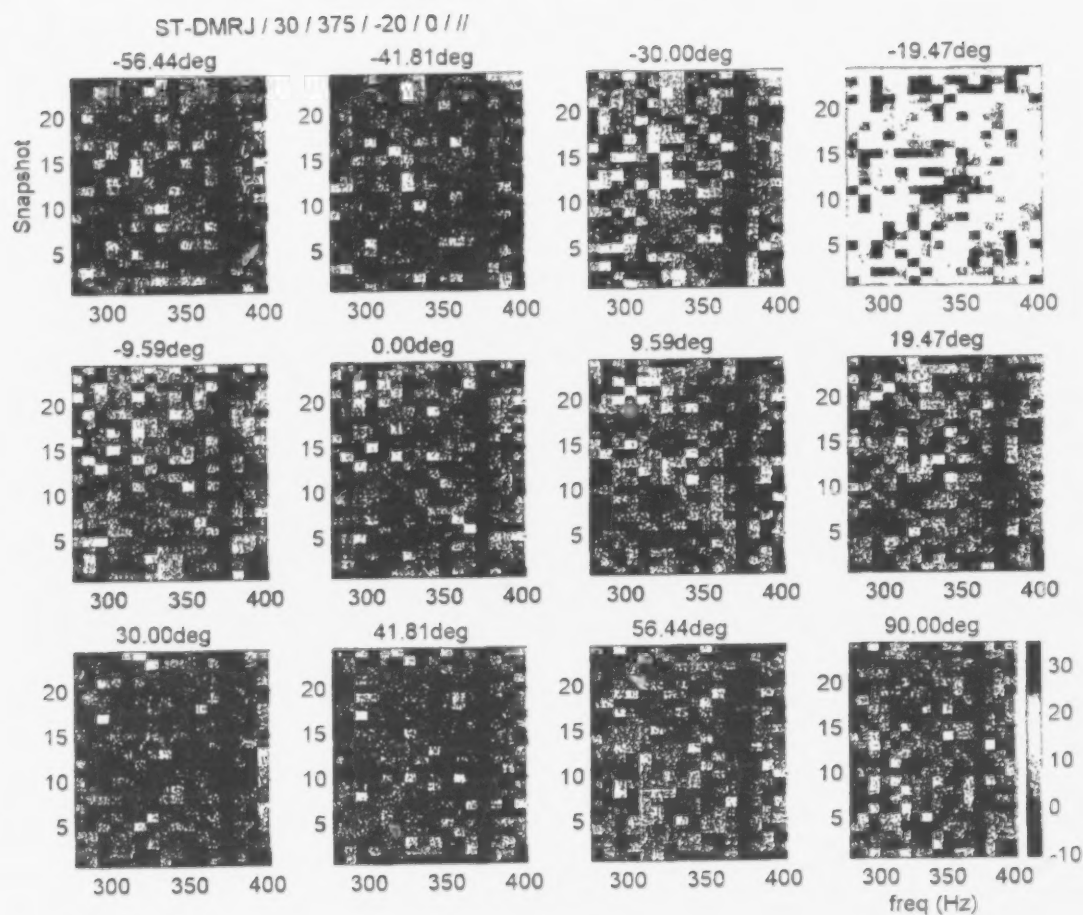


Fig. 5.1 ST-DMRJ beamformer output ($D=1$)

Tonal source at -20° , 375Hz. Expected output SNR = 30dB $N=12$ sensors Spacing $d=1\text{m}$ Processing band 280-400Hz Sampling rate 1kHz Snapshot length = 128 points 12 beams. 24 snapshots total Colorbar scale is in dB

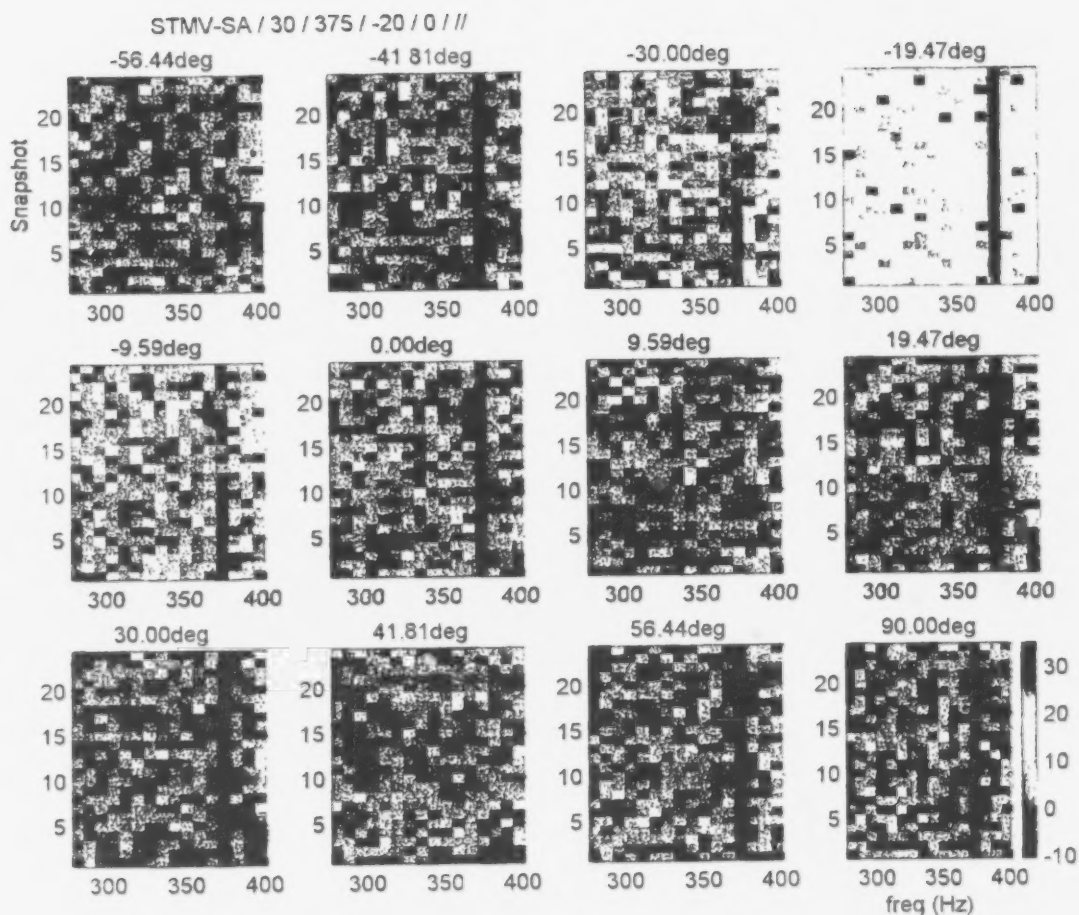


Fig. 5 2 STMV-SA beamformer output ($N_a=5$)

As Fig 5 1 but now note presence of strong line at -19.47° , 375Hz, as expected

6. SIMULATION COMPARISON OF VARIOUS TECHNIQUES FOR BEAMFORMING WITH SHORT OBSERVATION TIMES AND MISSING SENSORS

6.1. Introduction

We are now at the point where it is useful to compare the performance of the various techniques via simulation methods. A fairly comprehensive MATLAB program was written to perform such simulations which are all based on the processing of a series of consecutive (but non-overlapping) time-series snapshots. The processing is of the STCM-type described in earlier chapters. The simulations incorporate tonal signals (the narrowband, or passive, case) and/or chirp signals (the wideband, or active, case) in background noise. The noise can be uncorrelated or correlated, but in the examples shown here we use uncorrelated background noise.

As described in detail earlier, each time-series snapshot is decomposed via FFT processing into its Fourier coefficients, one set per sensor, which are collected to form Fourier coefficient vectors, one vector for each FFT bin. Each vector is then pre-steered, by phasing, to a chosen direction. After pre-steering, all of the vectors are used in calculating the single set of adaptive weights for that steering direction, or beam. The weights are applied to the original (steered) Fourier coefficient vectors to generate the overall estimates of the Fourier coefficients (one per FFT bin) which may be displayed directly to form a narrowband power map, or used to estimate the time-series output for that snapshot and that steering direction (or beam). In the latter case the snapshot outputs can be concatenated to form a longer time-series which can be fed through a matched filter (i.e. correlator) to form the overall WB time-series output.

In this chapter we examine the performance of the following beamformers:

- (a) CB-RECT. Conventional beamformer with uniform shading.
- (b) CB-HAMM. Conventional beamformer with Hamming shading.
- (c) ST-DMRJ. Steered dominant-mode rejection beamformer.

(d) STMV-SA. Steered minimum-variance subaperture method of Dhanatawari *et al* [2-2, 2-3, 2-4].

(e) STMV-SX. As STMV-SA but with a randomized subarray geometry.

(f) ALPINEX. Method involving aperture extrapolation (and/or interpolation) of Swingler and Walker [6-1].

The first two methods are non-adaptive; the second three are adaptive; the last is a hybrid mixture of adaptive and non-adaptive processing which was detailed in a DREA Contractor's Report in 1987 [6-1]. The three beamformers of especial interest in this report are (c), (d) and (e). Originally, the remainder were to be included primarily for comparison purposes. However, the examples that follow will demonstrate that the ALPINEX method remains a strong contender, especially where there are missing sensors.

Finally, we again remark it should be firmly grasped that, for a given method with a given observation interval and a given beam, the same weights are used across all frequency bins.

6.2 Simulation parameters

For the purposes of the simulation examples, the following basic scenario is established:

- (i) $N = 12$ sensor uniform line array
- (ii) $d = 1\text{m}$ inter-sensor spacing
- (iii) 1kHz sampling rate
- (iv) $M = 128$ time-series samples per snapshot (observation)
- (v) Processing band 280-400Hz
- (vi) Uncorrelated background noise

- (vii) Weights always normalized for unity gain in the pre-steering direction
- (viii) Twelve pre-steering directions (beams) equi-spaced in $\sin(\text{bearing})$ terms.
- (ix) Twenty-four consecutive snapshots processed with no overlap
- (x) Each snapshot is processed independently. No "memory" is retained of previous snapshots. This represents true "instantaneous convergence" for the adaptive processors.
- (xi) For each experiment, the signal and noise components are identical for each beamformer but both the noise and signal realisations change between experiments.

Unless otherwise specified the following details apply to the individual beamformers:

ST-DMRJ: $D=4$ dominant modes.

STMV-SA: Five subarrays ($N_a = 5$) with a subarray length of eight sensors ($N_s = 8$) and a one-sensor overlap (offset = 1) between adjacent subarrays. This yields $\text{DOF} = 4$. In the notation of Chapter 2, this is the STMV-SAI method.

STMV-SX: Five subarrays with randomized element locations. The randomization was based on an expected value of 75% of the array elements being randomly "selected". That is, 75% of the entries in the $\mathbf{K}$ matrix of Chapter 2 were expected to be 1's, the other 25% (on average), 0's. Again $N_a = 5$, so $\text{DOF}=4$. The same randomized $\mathbf{K}$ matrix was used throughout. This method was preferred in practice to the SAIM method of Chapter 3.

ALPINEX: Linear-prediction coefficients (order = 4) are calculated for each Fourier vector for each snapshot and are used to extrapolate the Fourier vectors equally in each direction to generate an overall array length of 36 sensors. Thereafter a 36-point Hamming shading vector is applied. "Unity gain" refers to the 36 Hamming weights, overall.

6.3 Tonal Experiments.

6.3.1 Two tonals at fixed bearings.

We begin by asserting that a simple scenario of interest is one where there are two tonals - one "strong", one "weak". This case is interesting as it highlights, at least in conventional beamforming, the possibility of the weaker tonal signal's being obscured by the sidelobes of the stronger. To quantify the notion of "strong" and "weak" we invoke the notion of a nominal output SNR measured "in the bin" and "in the beam" for a tonal signal in uncorrelated input noise as processed by an unshaded conventional beamformer (i.e. CB-RECT). This nominal SNR level is defined as

$$SNR = \frac{A^2 MN}{4\sigma^2} \quad (6.1)$$

where the input signal at any sensor is a real sinewave of amplitude, A , in real white noise of variance, σ^2 . The subsequent FFT processing decomposes the sinewave into two complex cisoids each of amplitude $A/2$ or power $A^2/4$. The M and N terms in the numerator represent the gains due to the time-series FFT and the CB-RECT beamformer, respectively.¹ It was found that in the experiments performed here, signals were barely detectable in the SONARGRAMS (by eye) at a nominal output SNR of 5dB ($= 10 \log_{10}(SNR)$). So we choose a nominal output SNR = 8dB for our weak signal and a much more powerful signal with nominal output SNR = 30dB for the stronger one. Initially, we fix the strong signal at bearing $\theta = -20^\circ$ and a frequency of 375Hz; the weak one at 0° , or broadside, and 320Hz.

Fig. 6.1(a) shows the NB power maps from the twelve beams for the CB-RECT beamformer. The twelve maps are arranged with the most negative bearing (-56.4°) in the upper left corner with the most positive ($+90^\circ$) in the lower right. The beam angles are indicated directly above each map. The output powers are designated in dB by colors according to the colorbar also in the bottom right. For scaling purposes, the extreme bottom left pixel in each map is set to a fixed maximum level (brownish-red) and the bottom right is set to a fixed minimum level (dark blue). In each map, time (in snapshots)

¹The input SNR is conventionally quoted as $A^2/2\sigma^2$.

is measured along the vertical axis, while frequency (in Hz) runs along the horizontal axis.

In the CB-RECT output of Fig. 6.1(a) we see a strong response (red line) at -19.5° , 375Hz with strong sidelobes in the adjacent beams at -30° and -9.6° (actually these are part of the mainlobe response). Weaker sidelobes (yellow lines) are clearly visible at 375Hz at -56.4° and 9.6° , 30° , 56.4° . The beams in between happen to fall at nulls in the CB-RECT beampattern for the strong tonal. Turning to the weak signal, we discern a fairly clear line at 320Hz, 0° as expected, with the hint of sidelobes in immediately adjacent beams.

In the CB-HAMM output of Fig. 6.1(b) we see a somewhat different response to the strong tonal at 375Hz. It is strongest at -19.5° as expected, but the response drops off rather slowly over the adjacent three beams and is thereafter not visible. This is consistent with the broader mainlobe, but much lower sidelobe levels of the CB-HAMM beamformer. The weak signal is again visible at 320Hz and 0° and in the immediately adjacent beams. The increase in the noise background level of about 1.3dB (relative to CB-RECT of the previous figure) is not readily discernible.

The ST-DMRJ results of Fig. 6.1(c) is as poor as predicted from our discussion in Chapter 5. The strong signal line is entirely missing from the beam at -19.5° and, due to the concomitant increase in weight norms, the background level at all frequencies in that beam are strongly elevated. In the neighbouring beams we note the presence of a strong null at the signal frequency of 375Hz. Though this is not regarded as a critical flaw, it is a central tenet of beamforming that all artifacts are to be avoided whenever possible.² There is barely any trace of the weak signal at 0° , 320Hz. This result is viewed as being so poor that no further use will be made of the ST-DMRJ beamformer in these experiments.

Moving onto the STMV-SA result in Fig. 6.1(d), we note that the strong line at 375Hz, -19.5° is accompanied by an elevated background level at all frequencies. This is due to the significant finite-data misadjustment effects connected with such a strong

²These "noise-nulls" are another consequence of finite-data processing in adaptive beamforming. They are also visible on the STMV-SA plots shown later. They do not appear to have been mentioned by other workers.

signal.³ (We recall that all the frequency components for a given snapshot are processed via a single WB covariance matrix). Also visible are deep nulls at 375Hz in every alternate beam. This is another aspect of the finite-data phenomenon. For the weak signal, we see a response in the beam at 0° as expected but we also note a slightly stronger response at 320Hz in the beam at -9.6°; an unwanted effect.

Finally, we turn to the ALPINEX results of Fig. 6.1(e) where we note that there is a strong tonal visible at 375Hz, -19.5° with sidelobes visible in the immediately adjacent beams, *only*. The weak signal is clearly visible at 320Hz, 0° as desired. The background level is lower by about 5dB than that of CB-RECT as expected from a unity gain ALPINEX beamformer with a threefold extrapolation factor [6-1].

Summary: Over all of the results in Fig. 6.1, it would seem that the ALPINEX beamformer is superior.

6.3.2 Two tonals at fixed bearings but with missing sensors.

One of the areas where the adaptive beamformers may be expected to offer superior performance occurs where there are (known) missing sensors. The adaptive beamformers simply adapt with relatively little performance penalty. However, the CB beamformers' sidelobe patterns are severely compromised by the presence of missing sensors. The CB weights are set to zero at the missing sensor locations and the remainder are scaled to retain overall unity gain in the steering (beam) direction

In Fig. 6.2(a) we see the CB-RECT NB output for the previous scenario but where now the three sensors at locations [5 7 8] are missing. As expected, the strong signal at 375Hz produces strong artifacts at this frequency over all of the beams. The problem is not as noticeable for the weak signal which remains visible at 320Hz, 0°.

³For completeness it might be mentioned that such elevated background levels are also observed at -19.5° even if the adaptive weights are based on an ideal "infinite-time" covariance matrix. However, in the infinite-time case, the elevation is due to the sensitivity of the beamformer to the difference in the beam direction of -19.5° and the true signal bearing of -20°. In the infinite-time case the background elevation effect vanishes if the beam angle and the source direction coincide; however it does not vanish for the corresponding finite-time case where misadjustment is the dominant phenomenon.

In Fig. 6.2(b) we have the corresponding results for the CB-HAMM beamformer. It is largely similar to the CB-RECT results of the previous figure with strong signal artifacts over all beams.

In Fig. 6.2(c) we have the STMV-SA results in which we note there is a considerable reduction in the number of strong signal artifacts at 375Hz. However, there still remain strong artifacts in the beams at -56.4° , 9.6° , 41.8° . These were not discussed in the work of Stergiopoulos et al but they were predicted in Chapter 3 of this report in connection with missing sensors and an interferer with an effective spatial frequency which is a multiple of $2\pi/N_s$, i.e. an interferer at a location corresponding to a null in the beam pattern of the subarray(s) at the frequency of the interference. We also note the elevated background levels in the beam at -19.5° , and lesser elevations in background levels in those other beams containing strong signal artifacts. There is also the more minor issue of deep "noise-nulls" at 375Hz in the remaining beams.

In an effort to eliminate the sidelobe artifacts of the STMV-SA beamformer, we use a randomised subarray geometry in the STMV-SX results of Fig. 6.2(d). The strong sidelobes at 375Hz in the beams at -56.4° , 9.6° , 41.8° are now missing, leaving the strong signal line confined to the beam at -19.5° . However, we again note the elevated background levels in that beam, and the nulls at 375Hz in other beams. The weak signal is barely visible at 320Hz, 0° . Overall, it would appear that the STMV-SX beamformer is clearly superior to the STMV-SA beamformer in this application.

Finally, we include the ALPINEX result in Fig. 6.2(e). Here the linear-prediction coefficients were used to interpolate for the missing sensor data as well as to extrapolate the array. As mentioned in [6-1], in the presence of missing sensors, it is useful to perform an iterative procedure to improve the estimate of the linear-prediction coefficients. In order to minimise the computational load, only two iterations were performed. More detail on execution times will be given later in this Chapter.

In Fig. 6.2(e) we note a strong line at 375Hz, -19.5° with sidelobes at this frequency limited to the immediately neighbouring beams. There are no other artifacts related to the strong signal. The weak signal is visible at 320Hz, 0° , as desired.

Summary. Overall, in these experiments with missing sensors, the ALPINEX technique appears to be the method of choice.

6.3.3 Two tonals; missing sensors; one tonal's bearing changing with time

Here we add a new wrinkle to the previous scenario. The bearing of the strong signal now varies linearly over the course of the experiment from -20° to $+30^\circ$. The sensors at locations [5 7 8] are still missing.

In Fig. 6.3(a) we have the CB-RECT results. The weak signal is clearly visible as a line at 320Hz in the beam at 0° but the situation with respect to the strong signal is much more difficult to interpret. With the benefit of hindsight, it is not too difficult to recognise that an energy peak works its way across the beams beginning at snapshot #1 in the beam at -19.5° and ending up at snapshot #24 in the beam $+30^\circ$. However the presence of sidelobe artifacts would, in the absence of this prior knowledge, make the interpretation rather challenging. Certainly if one looks along the 375Hz line at, say, 9.6° , the dominant impression is of a tonal fixed in bearing at 9.6° being switched on and off at a slow rate.

The CB-HAMM results of Fig. 6.3(b) are perhaps not quite so cluttered, but the remarks of the previous paragraph are still largely valid.

The STMV-SA results of Fig. 6.3(c) are quite interesting. We start by noting that the weak signal is visible at 0° but is somewhat obscured by the localised elevated background levels that accompany the nicely compact signal peak (single red pixel) near snapshot #10 at 375Hz. On the other beams the progress of the strong signal is easy to discern, though maybe as much by the presence of the elevated and "smeared" background levels as much as position the true quasi-point response itself.

However, closer inspection reveals perhaps a more troublesome phenomenon. If we look along the top row of beammaps, there is a well-defined secondary peak moving across the beams. It is most clearly visible near snapshot #5 in the beam at -41.8° , and near snapshot #15 in the beam at -19.5° . This is simply the STMV-SA sidelobe artifact of Fig. 6.2(c) following the strong source across the beams. A similar artifact is also clearly visible moving upwards along the bottom row of beammaps. Finally, we also note the presence of strong nulls in all the beams at 375Hz.

These sidelobe artifacts are absent in the STMV-SX results of Fig. 6.3(d). However, the "smearing" effect of the localised elevations in background noise levels around the true peaks remains a significant problem.

In Fig. 6.3(e) we show the ALPINEX results. Here we see the signal peak quite clearly moving across the beams. There is little in the way of strong signal artifacts and the weak signal remains visible at 320Hz, 0° .

Summary: This set of results again shows the ALPINEX method to be the technique of choice for this application.

6.3.4 Two tonals; one tonal's bearing changing with time; both tonals at the same frequency.

We now briefly explore the above situation with a moving source but where both tonals are at the same frequency. This is recognized as the multipath situation and is known to be highly problematic for MVDR-based beamformers, usually resulting in significant signal cancellation. In our experiment, both signals are set to the same frequency of 375Hz. We also make them of equal, "medium", strength with a nominal output SNR of 17dB each. For simplicity, there are no missing sensors.

In Fig. 6.4(a) we have the CB-RECT results. To the untrained eye, it is not easy to discern a pattern consistent with the aforementioned scenario. The situation is not much better in the CB-HAMM results of Fig. 6.4(b).

Although Figs. 6.4(a), (b) might be considered rather difficult to interpret, the STMV-SA and STMV-SX results of Figs. 6.4(c), (d) are probably considerably worse. There is very little energy visible outside of the region near snapshot #10 in the beam at 0° . This of course is where the two signals coincide in bearing. Elsewhere there is evidence of significant signal cancellation typical of MVDR-based beamformers in a multipath scenario. For the STMV-SA method, Fig. 6.4(c), there remains considerable energy visible near snapshot #24 in beams 0° and 30° but this turns out to be due to the fortuitous nulling of one of the two sources by a sidelobe null of the subarray(s); in the absence of one signal, the MVDR cancellation effect is not possible. This effect does not occur in the STMV-SX results of Fig. 6.4(d).

These results should be contrasted with those of the ALPINEX beamformer in Fig. 6.4(e) where there is clear evidence of an energy peak at 375Hz moving across the beams starting near snapshot #1 at -19.5° and ending near snapshot #24 at 30° . A strong stationary line is visible at 375Hz at 0° though its apparent energy increases around snapshot #10 as the two sources coincide in this beam at this time.

Summary: Again, it would seem that the ALPINEX method is the superior performer in this application.

6.4 Wideband Experiments

6.4.1 Two chirps with dissimilar arrival times

The tonal signals are now replaced by two linear FM chirps. Each has a linear frequency sweep from 313 to 375Hz over a duration of 500 samples, or about four snapshots in our experiments. For each snapshot, and for each beam, the Fourier coefficients at each frequency over the processing band of 280-400Hz were estimated by the various beamformers as described earlier. An inverse FFT was then used to generate the WB time-series output for each beam and the outputs for consecutive snapshots were concatenated⁴ to form a longer time-series (greater than duration of the chirp) which was then passed through a filter, or correlator, matched to the chirp. The expected signal output is a sinc-like function with a width measured from peak to first zero of about $(375-313)^{-1}$ sec or about 15 time samples.

We use two chirps. The stronger chirp has an amplitude (measured at the sensor) of 0.33, originates from a bearing of 20° and has an arrival time commencing at time-sample number 245. The weaker chirp has an amplitude of 0.132, bearing -30° , arrival at time-sample 350. The background noise standard deviation (also measured at the sensor) is 0.5. Note that the difference in times of arrival of 105 samples is large compared to the expected response width (15 samples) but quite short compared to the chirp duration (500 samples).

⁴For simplicity no overlap was used so there may be some minor problems associated with circular convolution issues in the concatenated snapshots. This area is not addressed here.

In Fig. 6.5(a) we show the amplitude of the correlator output time-series where the CB-RECT beamformer is employed. As before, we display the output for twelve beams equi-spaced in $\sin(\text{bearing})$. To make matters more challenging, sensors [5 7 8] are again deemed to have failed. We see that the strongest sinc-like response does occur at time-sample 245 in the beam at $+19.5^\circ$ as anticipated. According to the scaling used here, its peak height should be about $(0.5 \times \text{duration} \times \text{amplitude}) = 0.5 \times 500 \times 0.33$ or approximately 83 units. This is close to the level observed. The weaker signal is present in the beam at -30° with a peak amplitude of about 35 units. This is close to the expected value of $0.5 \times 500 \times 0.132 = 33$ units. The noise level is expected to have a standard deviation of $(0.5 \times \text{duration} \times \sigma^2 / (N-3))^{1/2} = 3$ units, approximately. Again this is typical of the noise levels observed in Fig. 6.5(a).

Because of the missing sensors we also see a large number of artifacts on Fig. 6.5(a). For instance there is a peak of amplitude about 50 in the beam at -9.6° . In the absence of *a priori* knowledge, the presence of such artifacts makes it more difficult to successfully interpret this figure.

Moving onto the use of the CB-HAMM beamformer, we have the results of Fig. 6.5(b) where again we see a number of artifacts in all beams. The CB-HAMM results are probably somewhat easier to interpret than the CB-RECT results.

The STMV-SA results of Fig. 6.5(c) may be considered to be more successful, with the expected peaks in the beams at $-30, +19.5^\circ$. However, there are peaks near time 245 in the beams at -9.6° and $+56.4^\circ$ which are again linked to the sidelobe artifacts alluded to in the STMV-SA tonal processing of Fig. 6.2(c). We also note an increased level of background noise particularly in beams where there is a signal peak. Finally we note that the signal peak at 19.5° is a little higher than expected.

The STMV-SX results are plotted in Fig. 6.5(d) and, as expected, the spurious peaks in the beams at $-9.6, +56.4^\circ$ are absent; only the two primary peaks are now easily discernible, though each is somewhat lower than expected.

The ALPINEX results of Fig. 6.5(e) also clearly display the two primary peaks, but there are some low level artifacts, at time 245 in the beams immediately adjacent to $+19.5^\circ$.

Summary: In these experiments with two chirps with dissimilar arrival times, the edge goes to the STMV-SX beamformer, although the elevated noise levels might pose a concern, closely followed by the ALPINEX method.

6.4.2 Two chirps with similar arrival times.

We now revise the experiment so that the weaker chirp arrives at time 250, just five time samples after the stronger one. However, the bearings are unchanged, and remain relatively widely-spaced, so no great problem is anticipated.

The CB-RECT and CB-HAMM results of Fig. 6.6(a), (b) are as expected. Primary peaks are visible in the correct beams at the correct times but with artifact peaks spread across all the other beams due primarily to the effect of the missing sensors.

A much more interesting case is that of the STMV-SA beamformer as shown in Fig. 6.6(c). The noise levels seem raised everywhere, the sinc peak near time 250 in the beam at 19.5° is considerably attenuated and the other signal peak expected at time 250 in the beam at -30° is entirely absent! The possibility of this type of severe signal cancellation in the WB scenario does not seem to have been explicitly mentioned elsewhere. It is discussed in more detail in the next section.

The results of the STMV-SX beamformer are little improved (Fig. 6.6(d)). Again we see an attenuation of the strong peak in the beam at 19.5° and the expected peak in the beam at -30° is again missing. This tends to confirm the notion that there is an MVDR signal cancellation effect at play here.

Finally, we present the ALPINEX results in Fig. 6.6(e). The two primary peaks are indeed visible in the beams at -30 and $+19.5^\circ$, though there are some low level artifacts in the neighbouring beams in the immediate vicinity of the strong signal at 19.5° .

Summary: With similar arrival times, the STMV-SA and STMV-SX beamformers are subject to a severe signal cancellation effect. In this experiment, the ALPINEX beamformer is the beamformer of choice.

6.5 Discussion of the STMV signal cancellation effect in WB processing

Let us first consider tonal (NB) processing. It is well known that if two signals are co-located in time and frequency, then any NB-MVDR processor will perform cancellation on these signals. This is the multipath phenomenon. However, if the frequencies are separated by more than the reciprocal of the snapshot length, then this effect might be expected to be negligible.

If we apply the same notion to the WB case, we might expect there to be a cancellation problem if the instantaneous frequencies of two temporally overlapping chirps are separated by less than the reciprocal of the snapshot length. In our case, with linear FM chirps, we can use the approximation that the difference in instantaneous frequencies is given roughly by Δf where

$$\Delta f \approx \frac{\Delta F}{T} \tau \quad (6.2)$$

where ΔF is the overall chirp frequency sweep, T is the duration of the chirp and τ is the difference in arrival times. In order to avoid the signal cancellation effect, we must have $\Delta f > T_s^{-1}$ where T_s is the length of an individual snapshot. So overall, to avoid MVDR signal cancellation, we must have

$$\tau > \frac{T}{T_s} \Delta F^{-1} \quad (6.3)$$

In our experiment, expressing everything in samples, $T_s = 128$, $T = 500$, $\Delta F^{-1} = 15$ samples, so (6.3) implies $\tau > 60$ samples, approximately. In section 6.4.2, we had $\tau = 5$ samples and hence, in hindsight, we might have anticipated the significant cancellation problems observed with the STMV-SA, STMV-SX WB beamformers in such a scenario. In Section 6.4.1, however, we had $\tau=105$ and no signal cancellation was observed.

In passing, we might note that (6.3) states that the difference in arrival times must be greater than the output sinc width multiplied by the ratio of the chirp duration to the snapshot length. This is an interesting result with practical consequences.

6.6 A comment on processing times

In order to have some feel for the relative computational loads of the various methods, MATLAB's clock function was used to estimate the time taken to perform the pre-steering step and weight calculations for a given snapshot in a given beam. The basic ALPINEX method (without interpolation) was about 3 times slower than the STMV-SA method and about 4 times slower if the iterative interpolation routine was invoked. All the adaptive methods were at least an order of magnitude slower than the CB beamformers where the weight values are fixed.

If however the overall processing time was measured, including the plotting times for the NB and WB maps as shown in this chapter, then using the CB-RECT as a unit-time reference, the following relative times were observed CB-HAMM=1; STMV-SA(X)=1.8; ALPINEX(w/o Interpolation)=2.3; ALPINEX(with Interpolation)= 3.4.

The ALPINEX method is slowed by calculating separate prediction coefficients for each FFT bin. Although not attempted here, it should not be difficult to reduce its computational load by computing one set of prediction coefficients for the entire pre-steered covariance matrix in much the same way as block processing is performed for the other adaptive methods discussed in this report.

6.7 Conclusion

We have demonstrated that in multipath and non-multipath NB scenarios involving short line-arrays with missing sensors, the ALPINEX method is the method of choice. Typically, it minimises the sidelobe problems of the conventional beamformers while being insensitive to problems, such as the multipath effect, which beset the STMV-based beamformers.

In the WB scenario, we have demonstrated that the STMV-SA methods are susceptible to severe signal cancellation effects if the arrival times are not sufficiently dissimilar. To be more specific, to avoid cancellation effects, the arrival times must differ by at least the reciprocal of the frequency sweep multiplied by the ratio of the chirp length to the primary snapshot length. This result was not anticipated from the earlier

work in [2-1, 2-2, 2-3]. Again, in the WB scenario, the ALPINEX method remains overall the best choice in the applications discussed here.

On a note of lesser import, the STMV-SA method of Stergiopoulos et al, when applied to arrays with missing sensors, is susceptible to sidelobe-type artifacts linked to the underlying regular subarray geometry. This problem is largely ameliorated on using the STMV-SX beamformer where the subarrays have elements with randomised locations.

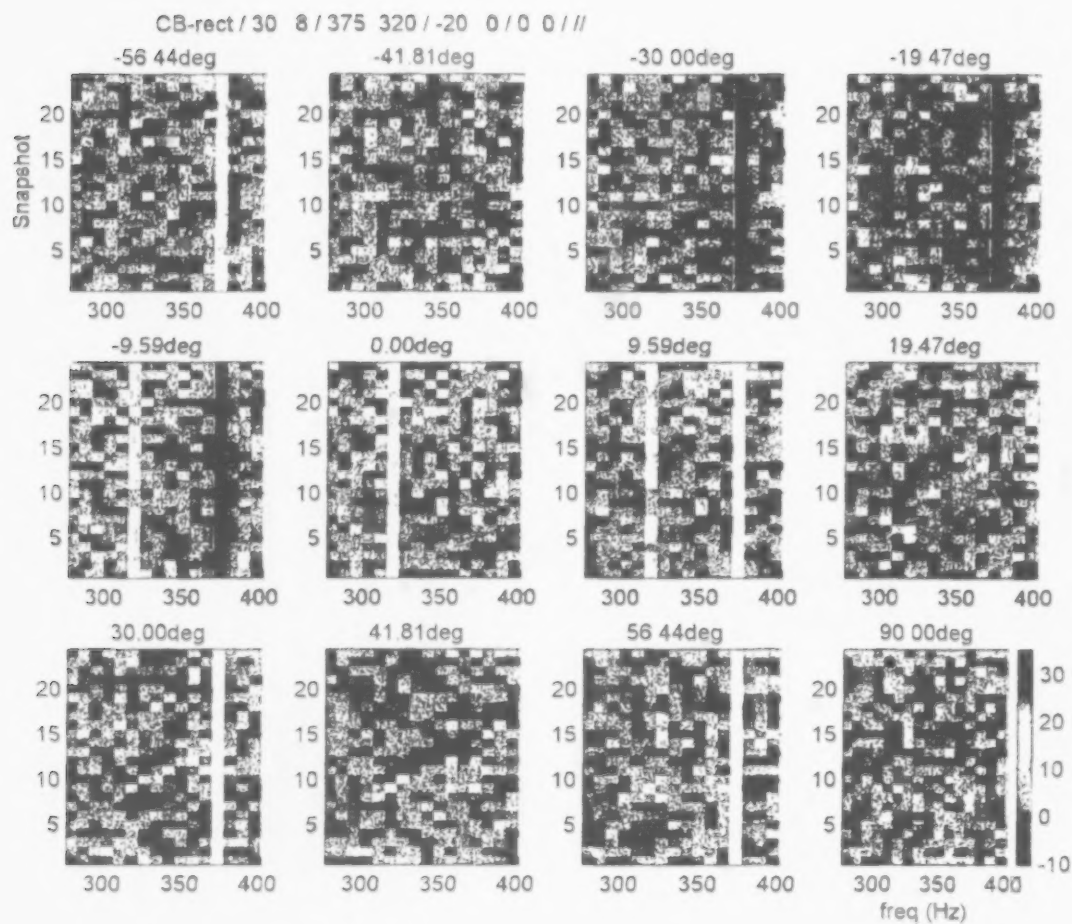


Fig 6.1(a) NB CB-RECT beamformer output

Two tonal sources -20° , 375Hz., nominal output SNR = 30dB; 0° , 320Hz., nominal output SNR = 8dB $N=12$ sensors. Spacing $d=1m$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. 24 snapshots total. Colorbar scale at bottom right is in dB

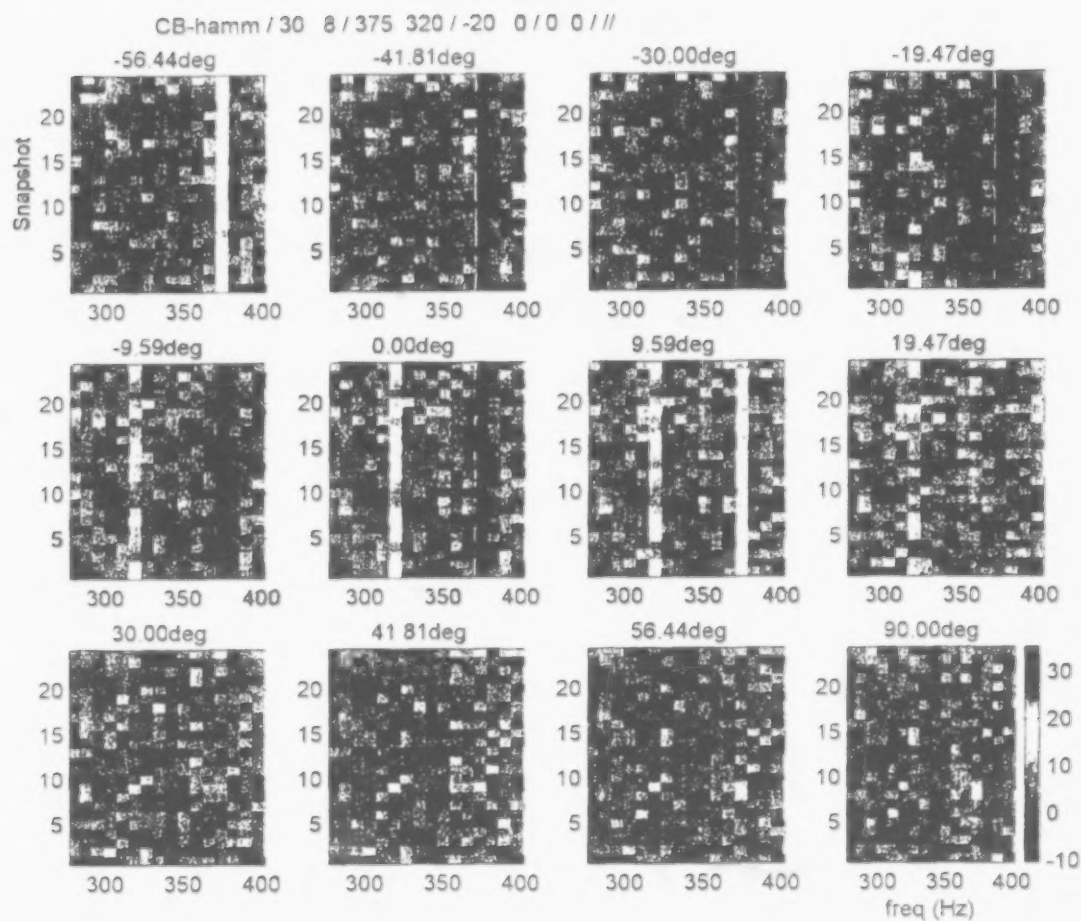


Fig. 6 1(b) NB CB-HAMM beamformer output

Two tonal sources: -20° , 375Hz., nominal output SNR = 30dB, 0° , 320Hz., nominal output SNR = 8dB. N=12 sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. 24 snapshots total. Colorbar scale at bottom right is in dB.

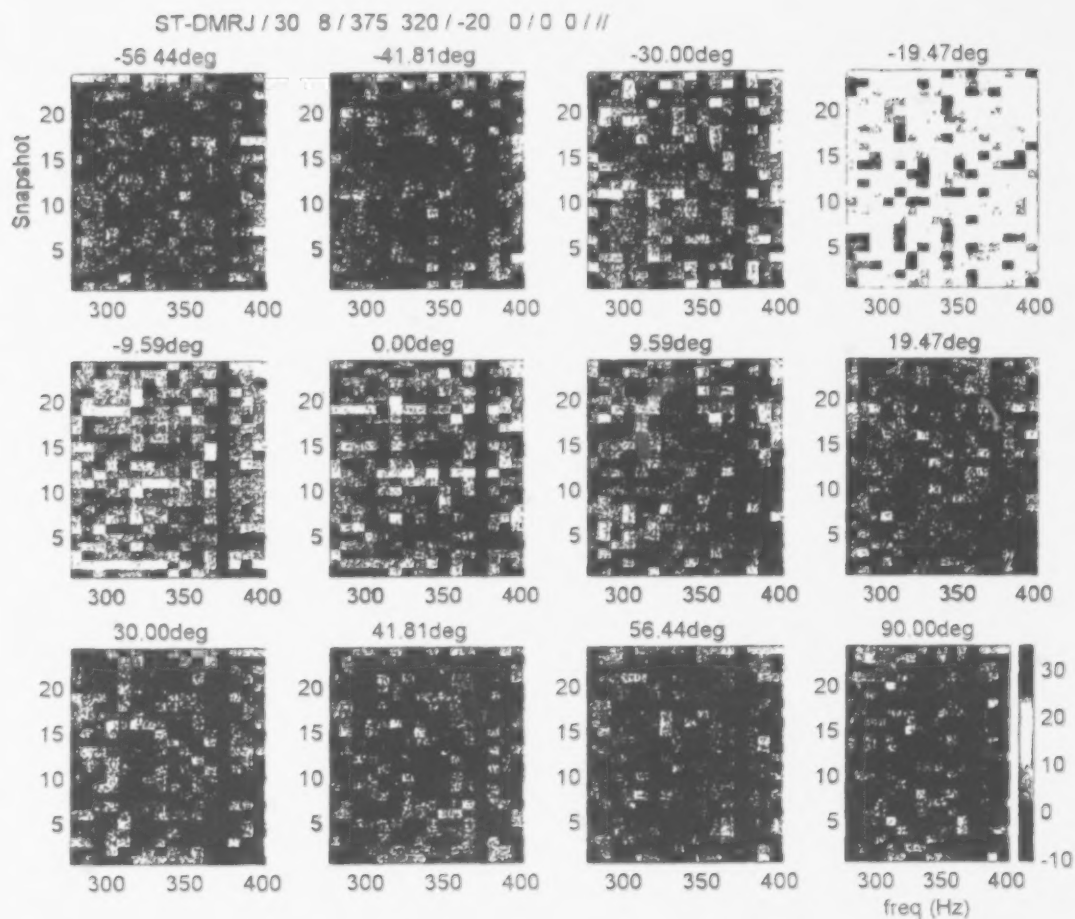


Fig. 6.1(c) NB ST_DMRJ beamformer output

Two tonal sources -20° , 375Hz., nominal output SNR = 30dB, 0° , 320Hz., nominal output SNR = 8dB N=12 sensors. Spacing $d=1\text{m}$ Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams 24 snapshots total Colorbar scale at bottom right is in dB

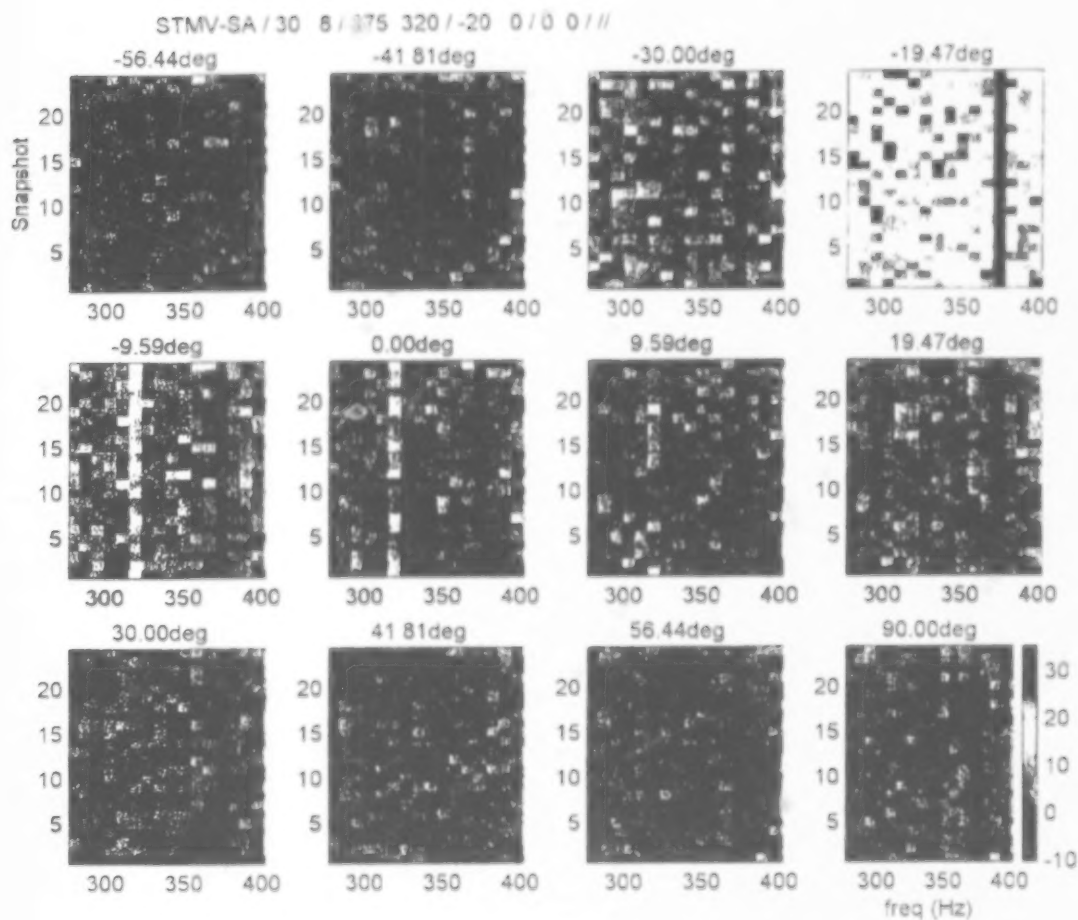


Fig. 6 1(d) NB STMV-SA beamformer output

Two tonal sources: -20° , 375Hz, nominal output SNR = 30dB, 0° , 320Hz, nominal output SNR = 8dB. N=12 sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. 24 snapshots total. Colorbar scale at bottom right is in dB.

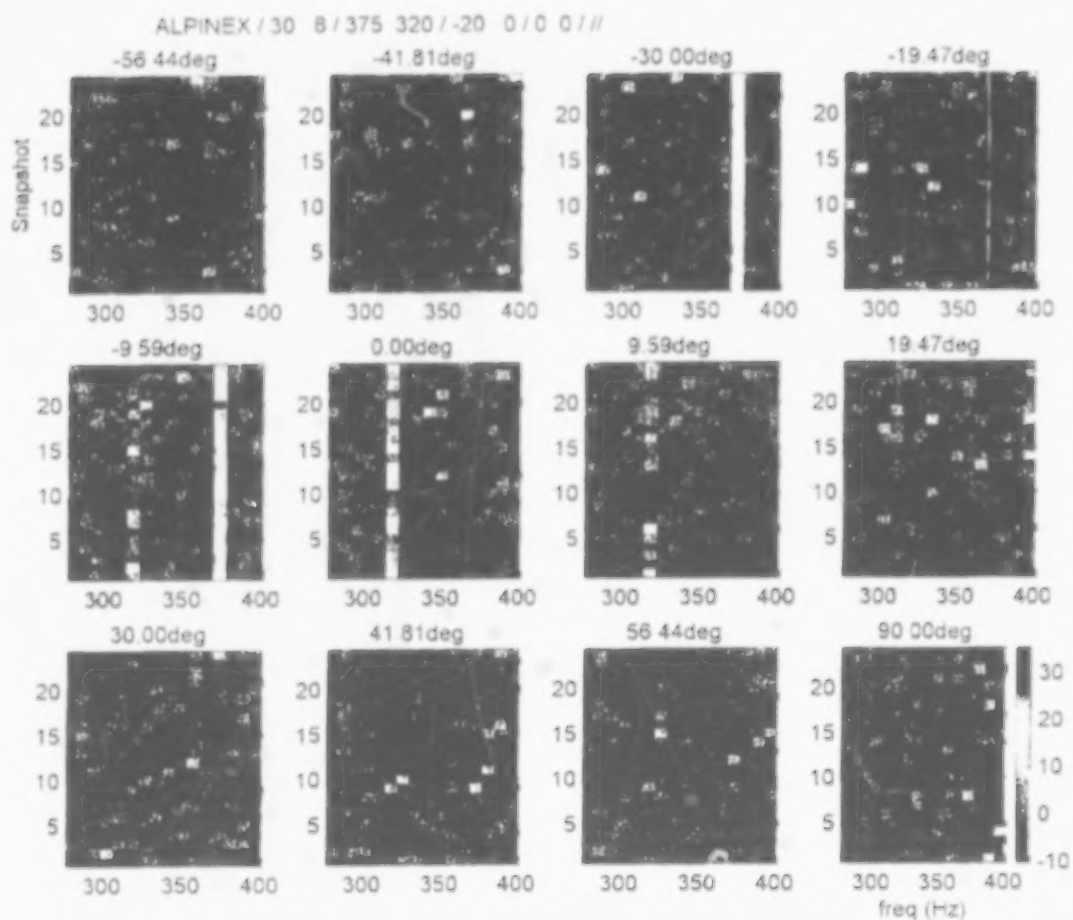


Fig. 6.1(e) NB ALPINEX beamformer output

Two tonal sources. -20° , 375Hz, nominal output SNR = 30dB, 0° , 320Hz, nominal output SNR = 8dB. N=12 sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. 24 snapshots total. Colorbar scale at bottom right is in dB.

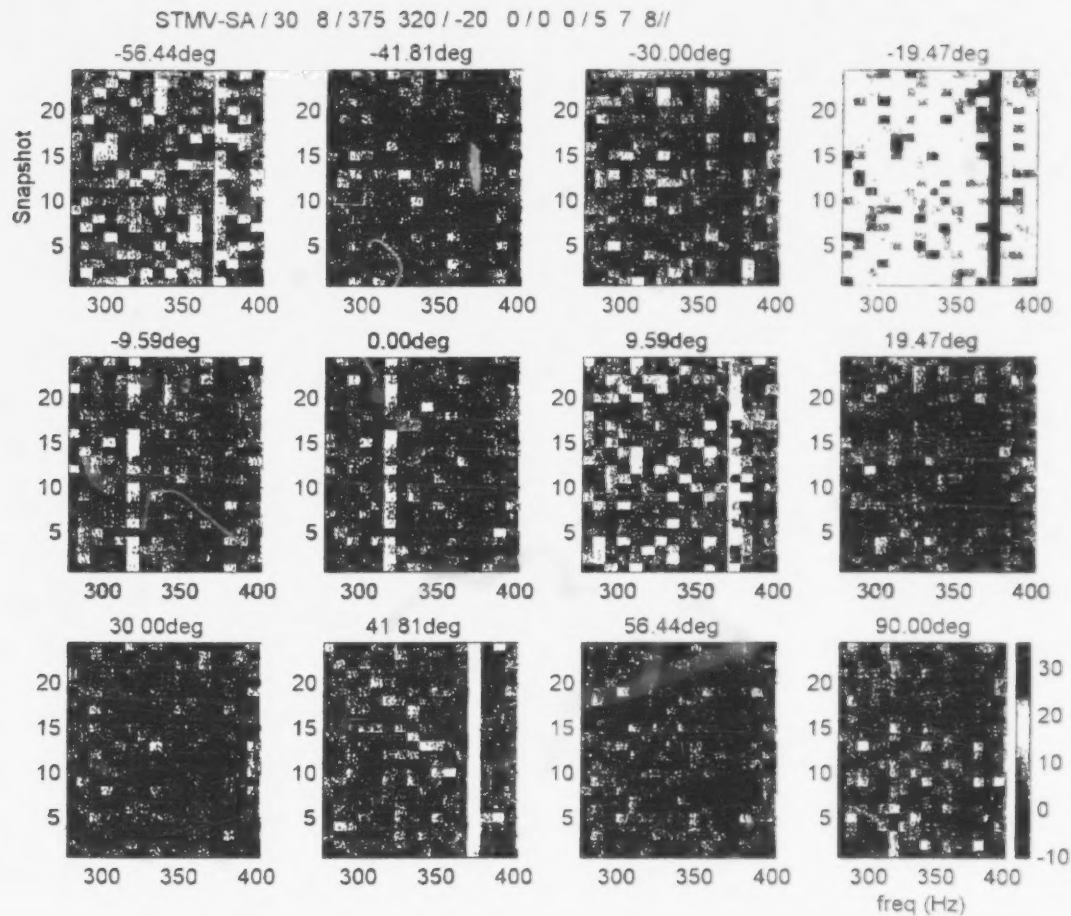


Fig. 6.2(c) STMV-SA beamformer output

As Fig. 6.1(d) but with sensors [5 7 8] missing

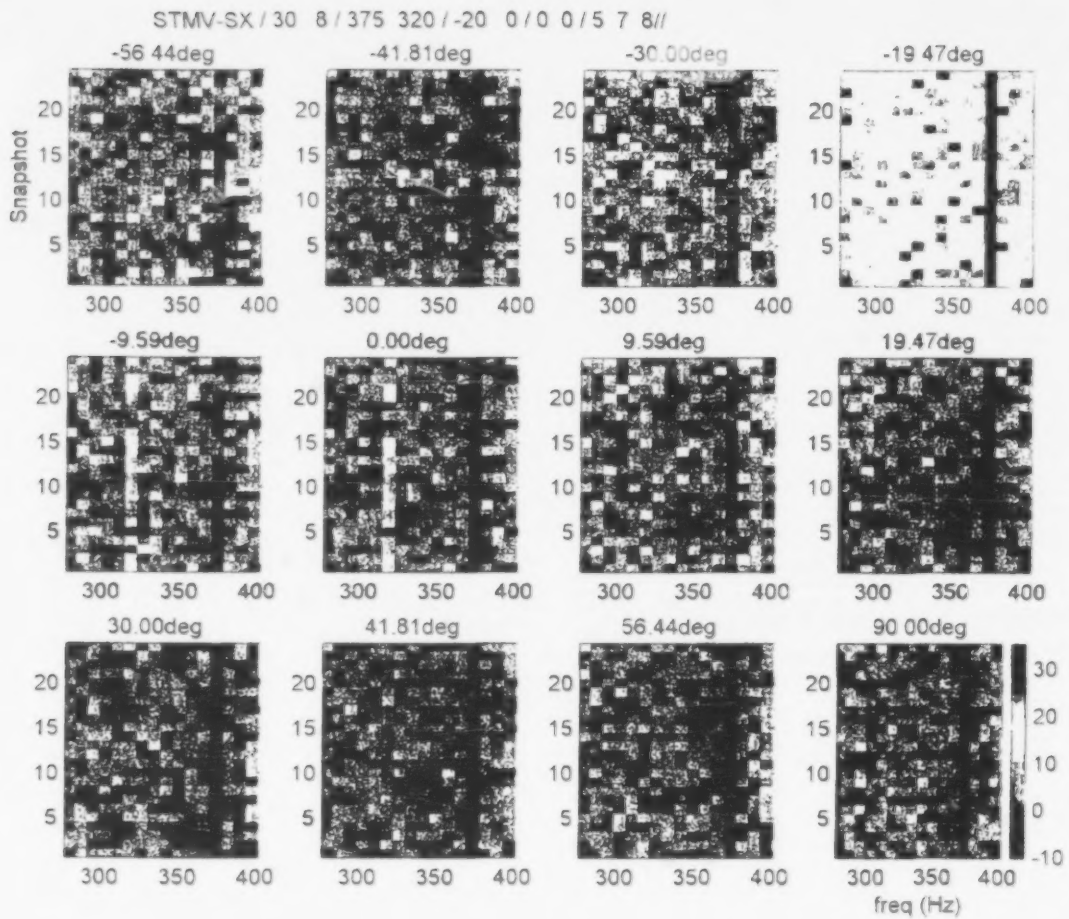


Fig 6 2(d) NB STMV-SX beamformer output

As Fig 6.1(d) but with sensors [5 7 8] missing and with modified subarray geometry



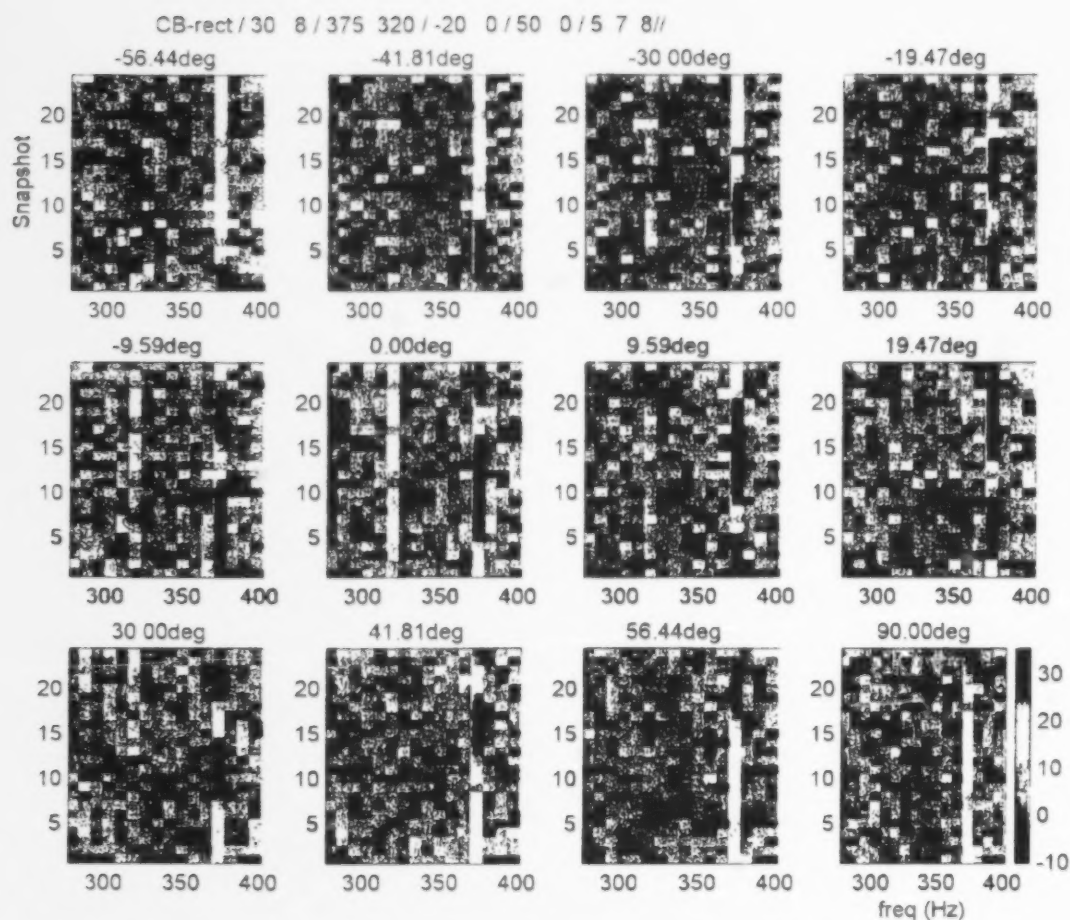


Fig 6 3(a) NB CB-RECT beamformer output with missing sensors and a moving source

As Fig 6 1(a) but with sensors [5 7 8] missing and the stronger source moving from -20° to $+30^\circ$

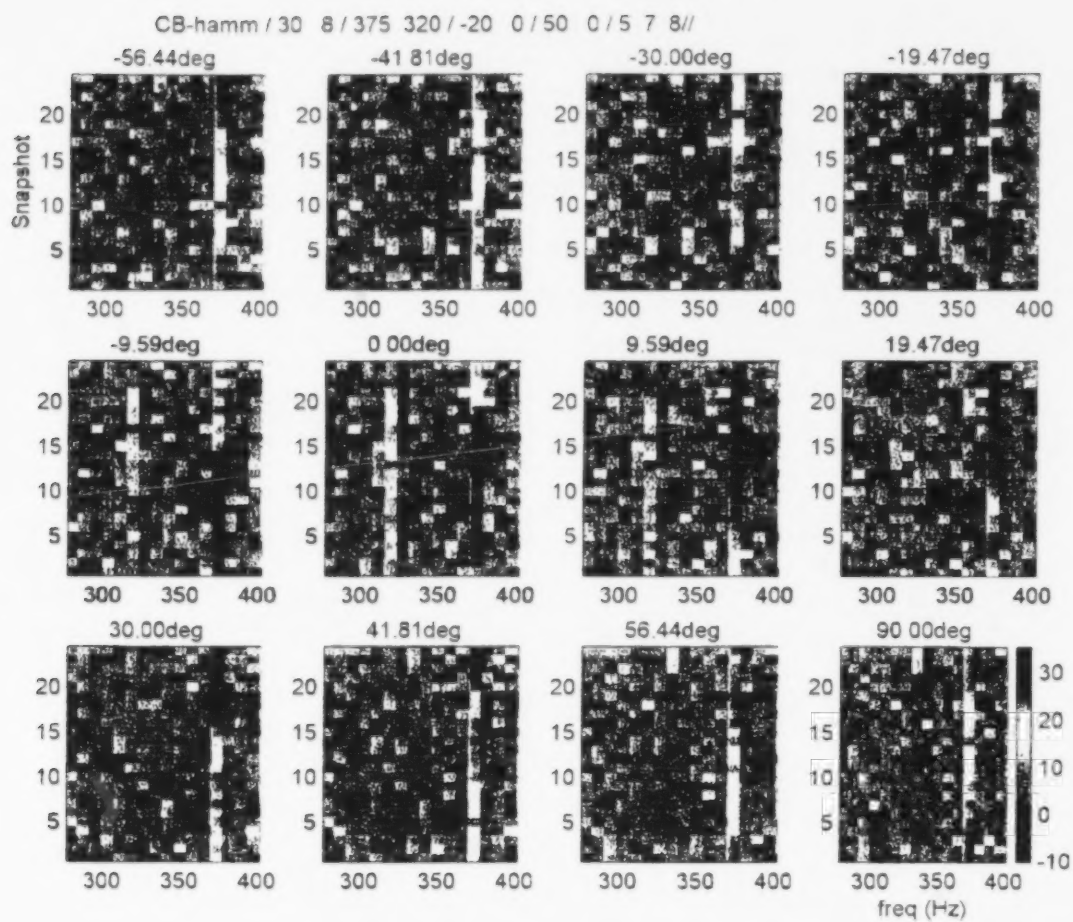


Fig 6.3(b) NB CB-HAMM beamformer output with missing sensors and a moving source

As Fig 6.1(b) but with sensors [5 7 8] missing and the stronger source moving from -20° to $+30^\circ$.

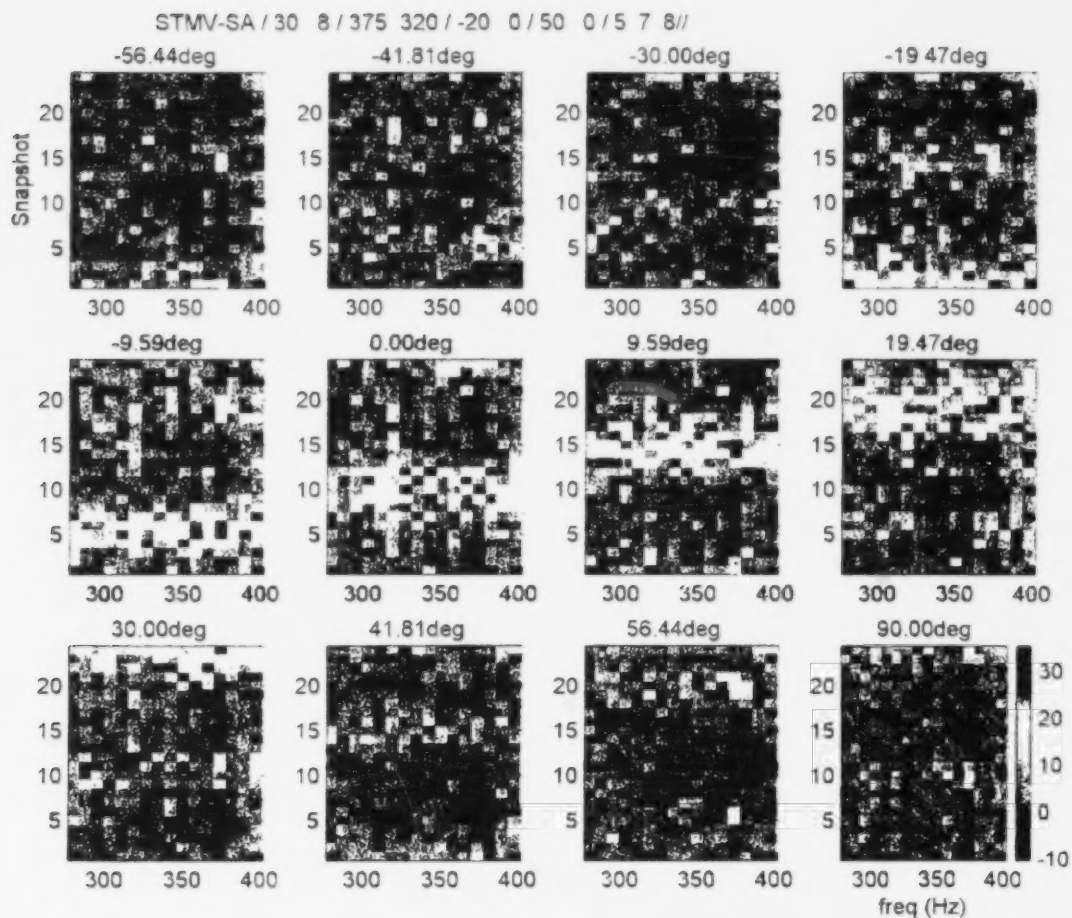


Fig 6 3(c) NB STMV-SA beamformer output with missing sensors and a moving source

As Fig 6.1(d) but with sensors [5 7 8] missing and the stronger source moving from -20° to $+30^\circ$

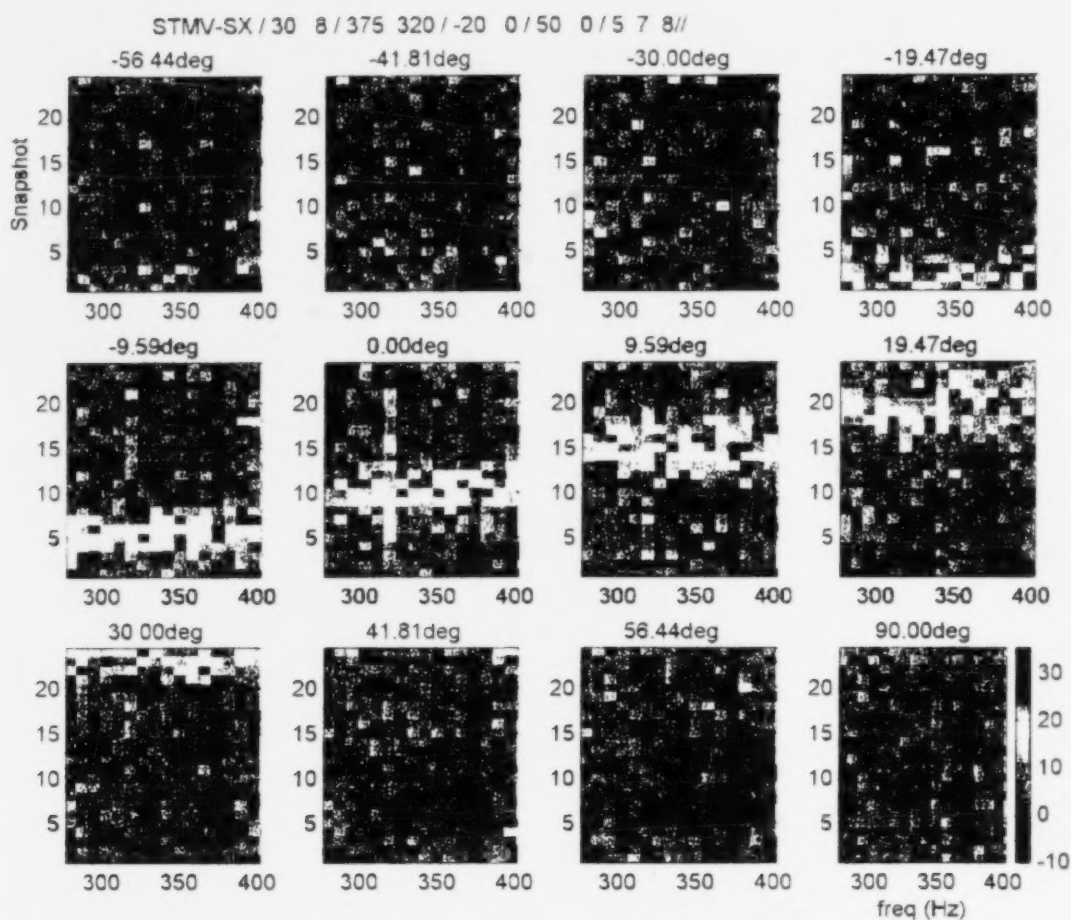


Fig. 6.3(d) NB STMV-SX beamformer output with missing sensors and a moving source

As Fig. 6.2(d) but with sensors [5 7 8] missing and the stronger source moving from -20° to $+30^\circ$.

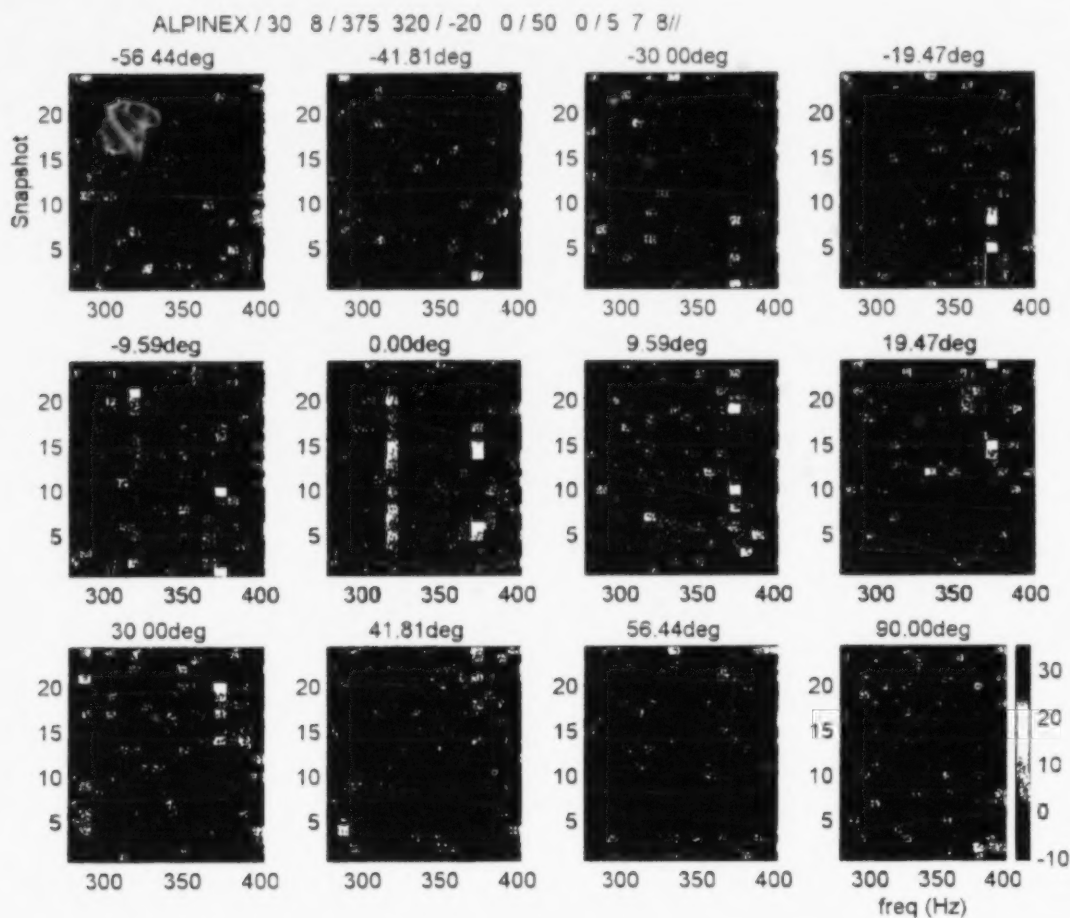


Fig 6 3(e) NB ALPINEX beamformer output with missing sensors and a moving source

As Fig. 6 1(e) but with sensors [5 7 8] missing and the stronger source moving from -20° to $+30^{\circ}$

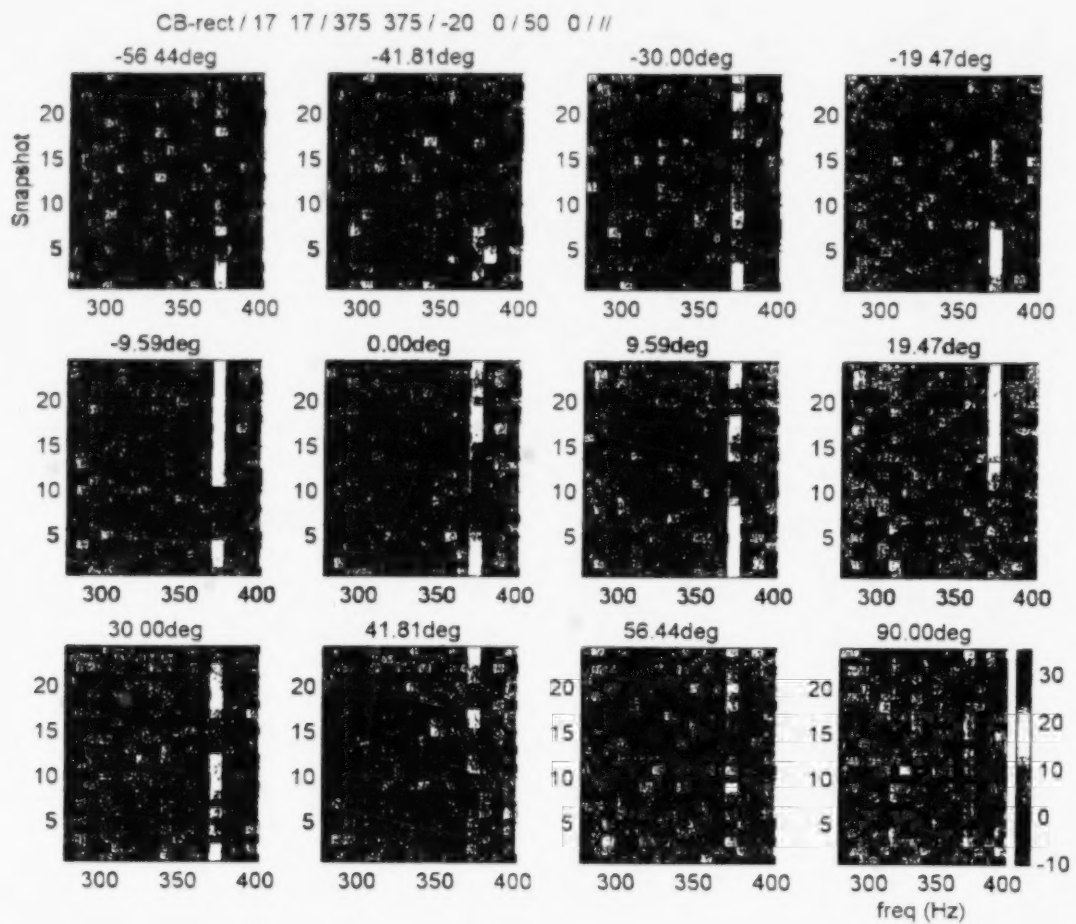


Fig. 6 4(a) NB CB-RECT beamformer output with two sources at the same frequency

As Fig. 6 3(a) but with no missing sensors and both sources at 375Hz

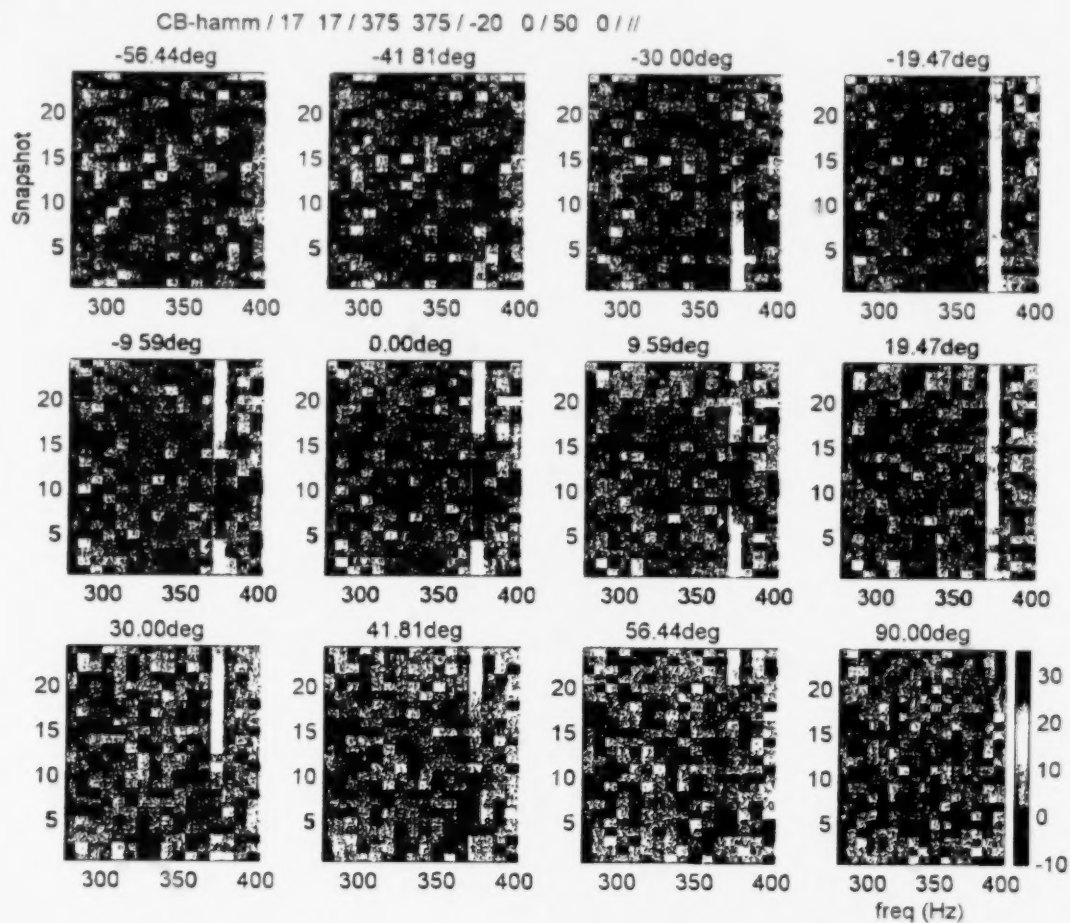


Fig 6.4(b) NB CB-HAMM beamformer output with two sources at the same frequency

As Fig 6 3(b) but with no missing sensors and both sources at 375Hz.

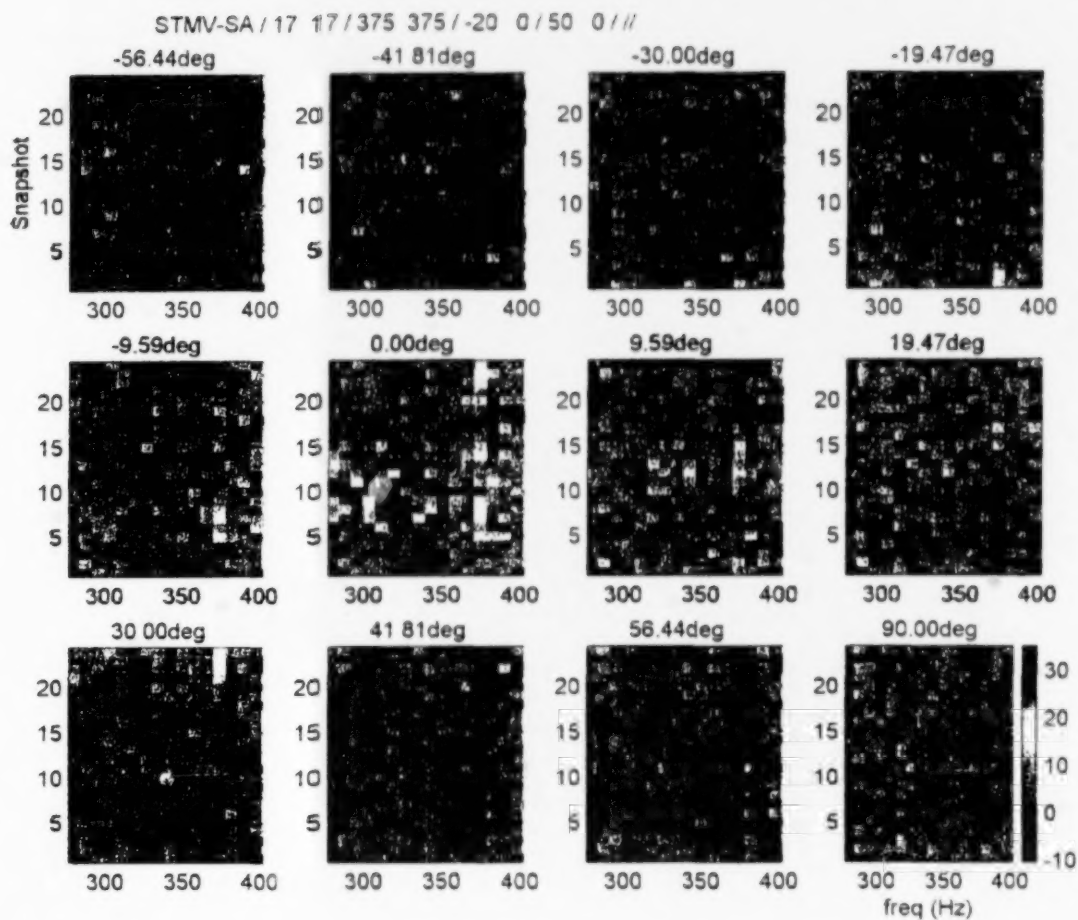


Fig 6 4(c) NB STMV-SA beamformer output with two sources at the same frequency.

As Fig 6 3(c) but with no missing sensors and both sources at 375Hz Note significant cancellation effects.

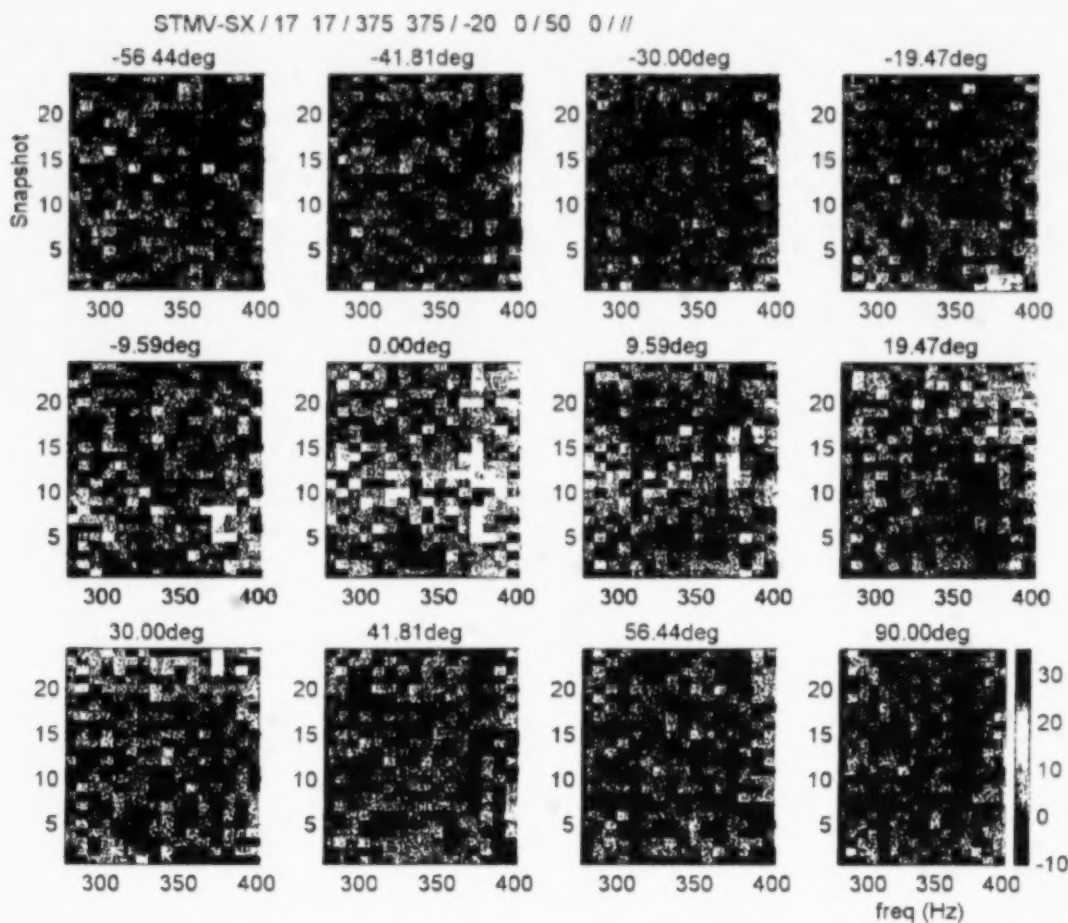


Fig. 6 4(d) NB STMV-SX beamformer output with two sources at the same frequency

As Fig. 6 3(d) but with no missing sensors and both sources at 375Hz. Note significant cancellation effects

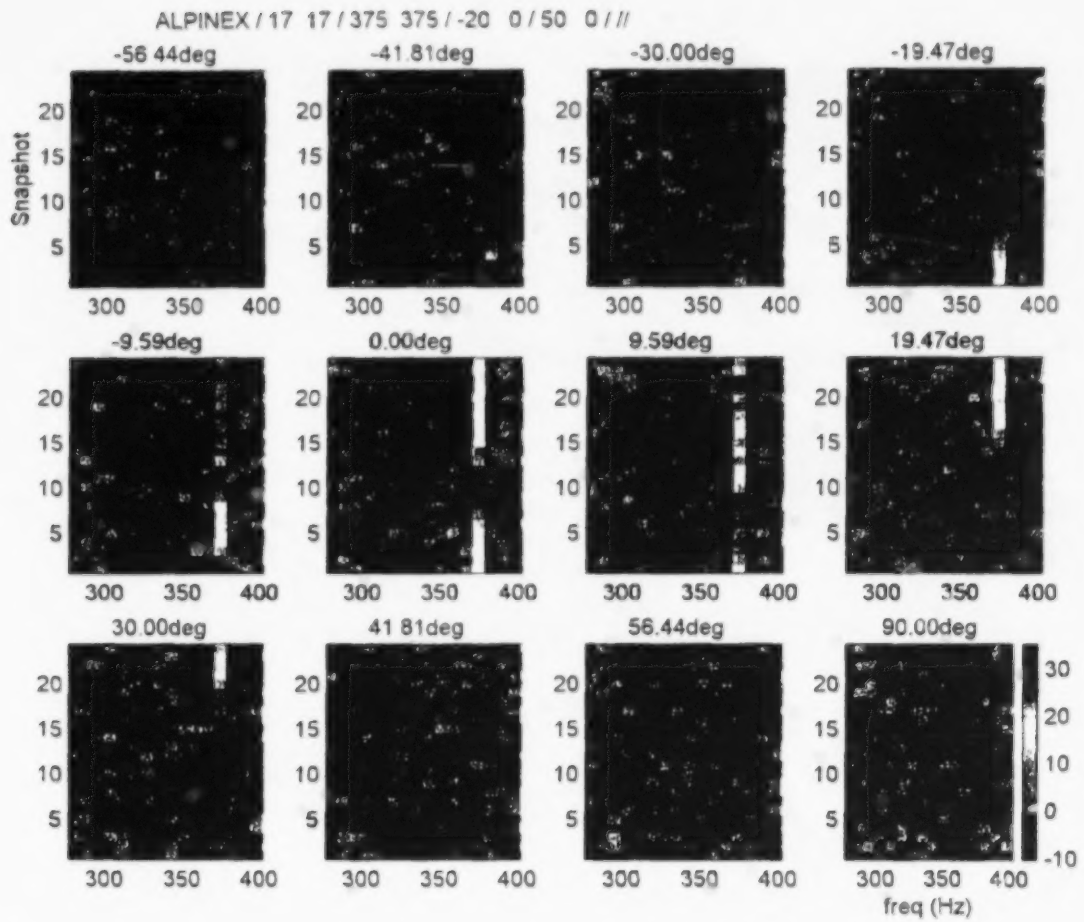


Fig 6 4(e) NB ALPINEX beamformer output with two sources at the same frequency

As Fig 6 3(e) but with no missing sensors and both sources at 375Hz

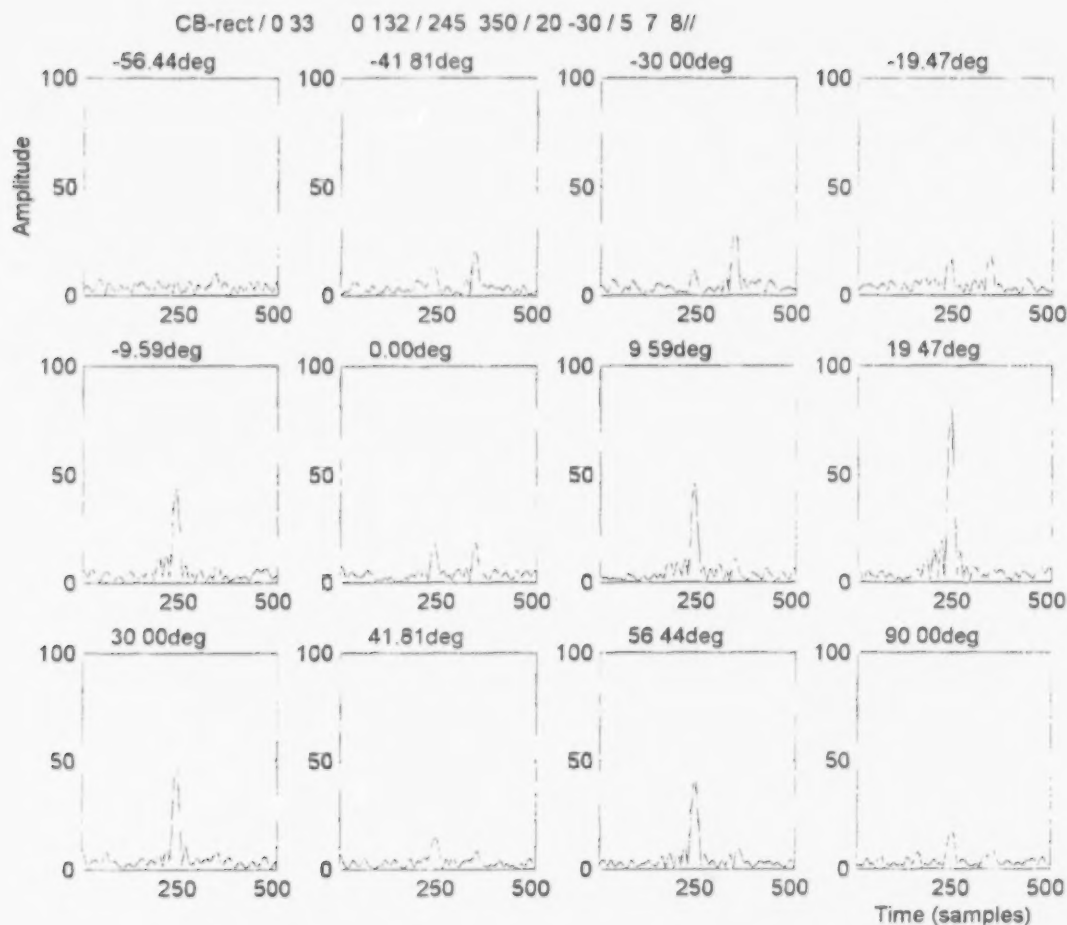


Fig 6 5(a) WB CB-RECT beamformer Correlator output for two chirps with dissimilar arrival times

Two chirp sources Stronger at 20° , arrival time at sample 245, weaker at -30° , arrival time at sample 350 Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples $N=12$ sensors. Spacing $d=1\text{m}$ Processing band 280-400Hz Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams Sensors [5 7 8] missing

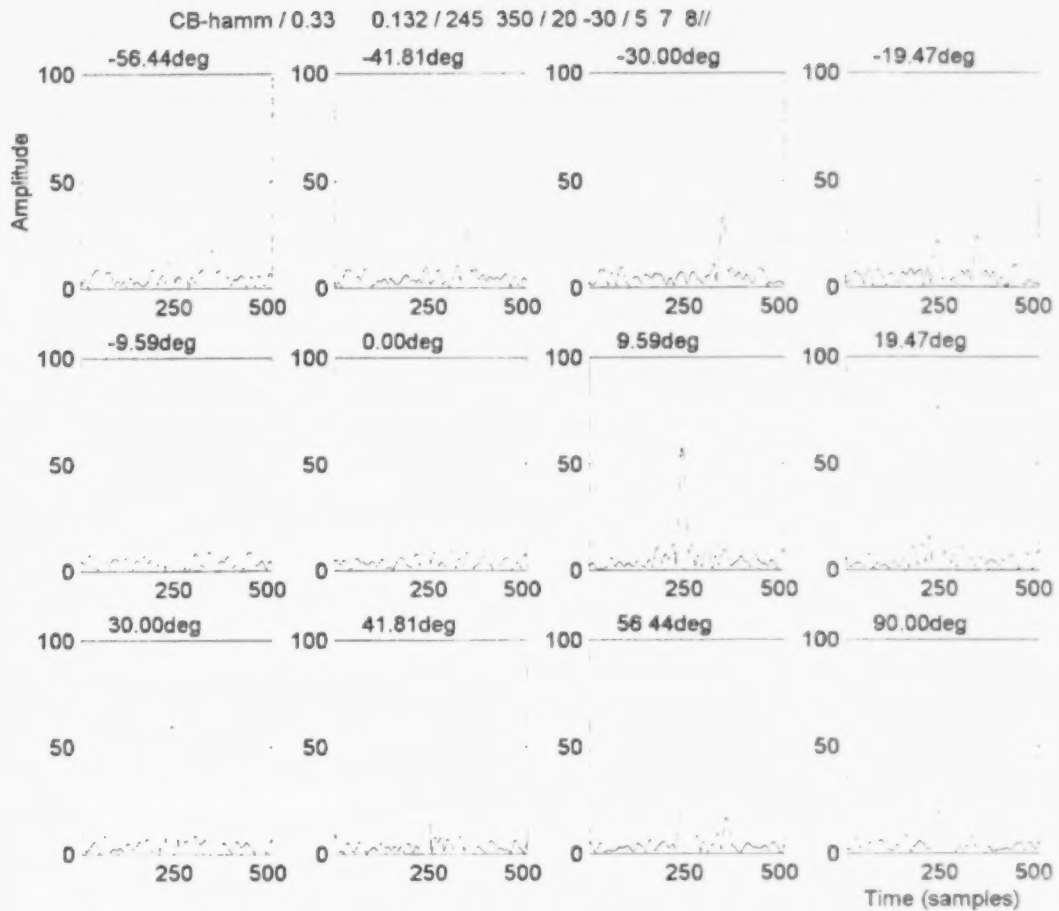


Fig. 6.5(b) WB CB-HAMM beamformer. Correlator output for two chirps with dissimilar arrival times

Two chirp sources: Stronger at 20° , arrival time at sample 245, weaker at -30° , arrival time at sample 350. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples $N=12$ sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. Sensors [5 7 8] missing.

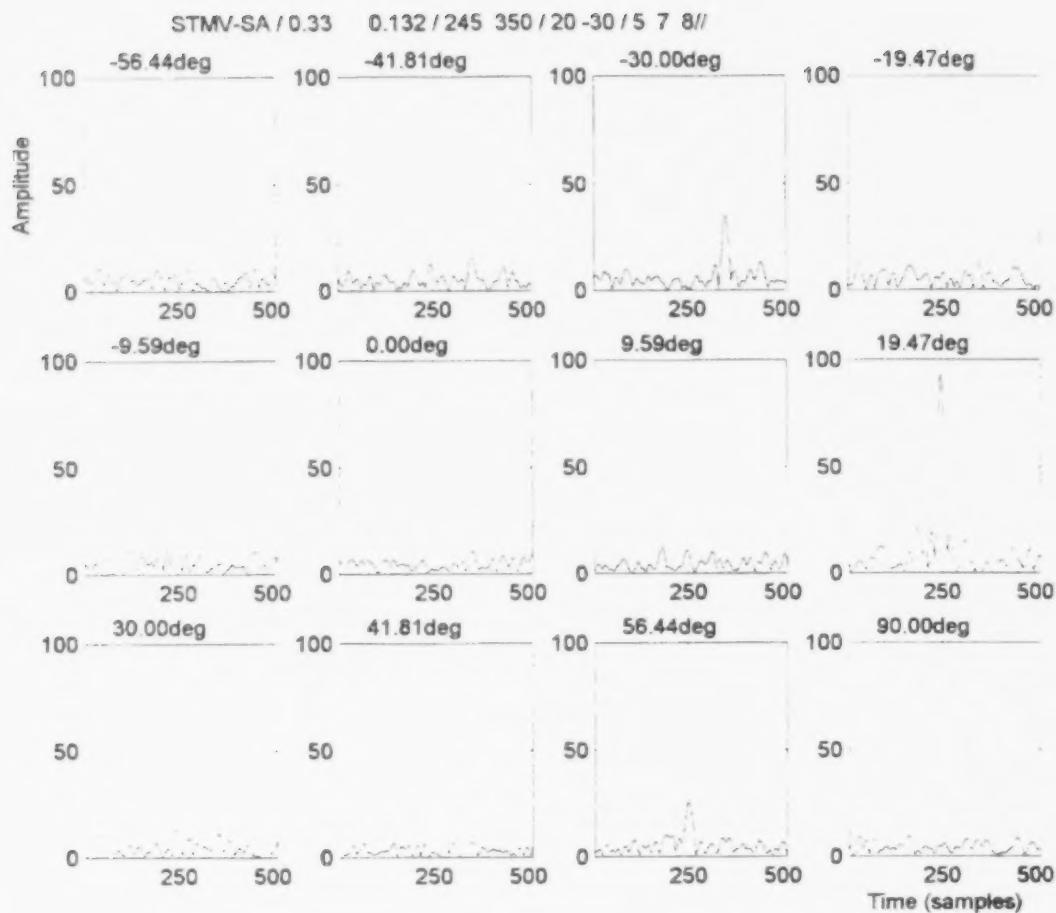


Fig 6.5(c) WB STMV-SA beamformer. Correlator output for two chirps with dissimilar arrival times

Two chirp sources. Stronger at 20° , arrival time at sample 245; weaker at -30° , arrival time at sample 350. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples. $N=12$ sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. Sensors [5 7 8] missing.

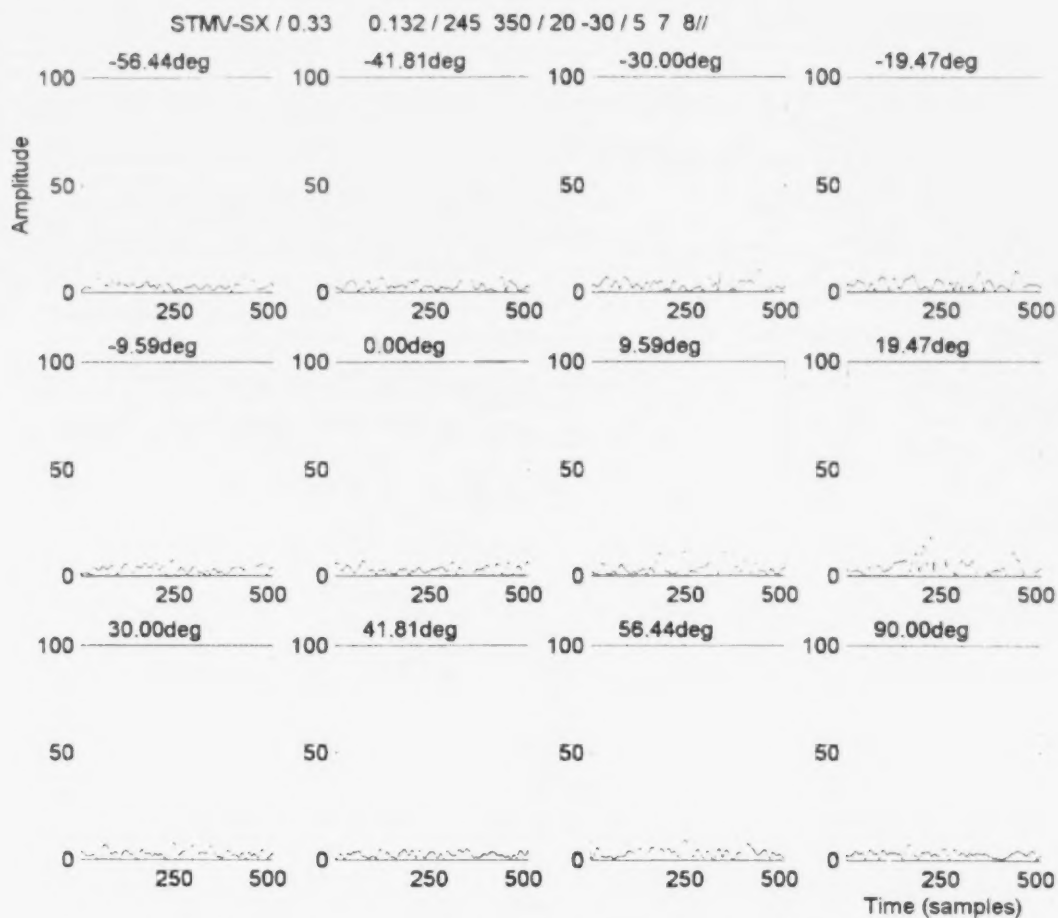


Fig. 6 5(d) WB STMV-SX beamformer Correlator output for two chirps with dissimilar arrival times.

Two chirp sources: Stronger at 20° , arrival time at sample 245; weaker at -30° , arrival time at sample 350. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples $N=12$ sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz Snapshot length = 128 time samples. 12 beams Sensors [5 7 8] missing.

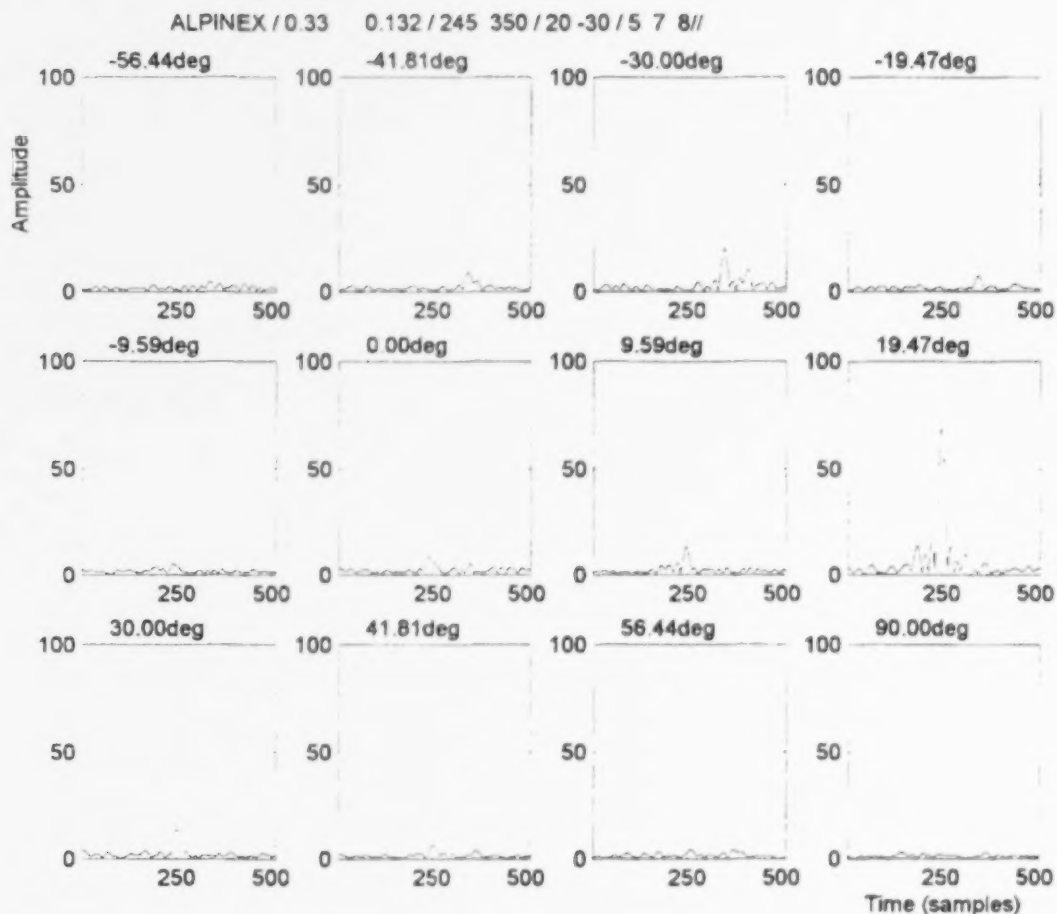


Fig 6.5(e) WB ALPINEX beamformer. Correlator output for two chirps with dissimilar arrival times.

Two chirp sources. Stronger at 20° , arrival time at sample 245; weaker at -30° , arrival time at sample 350. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples. $N=12$ sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples 12 beams. Sensors [5 7 8] missing.

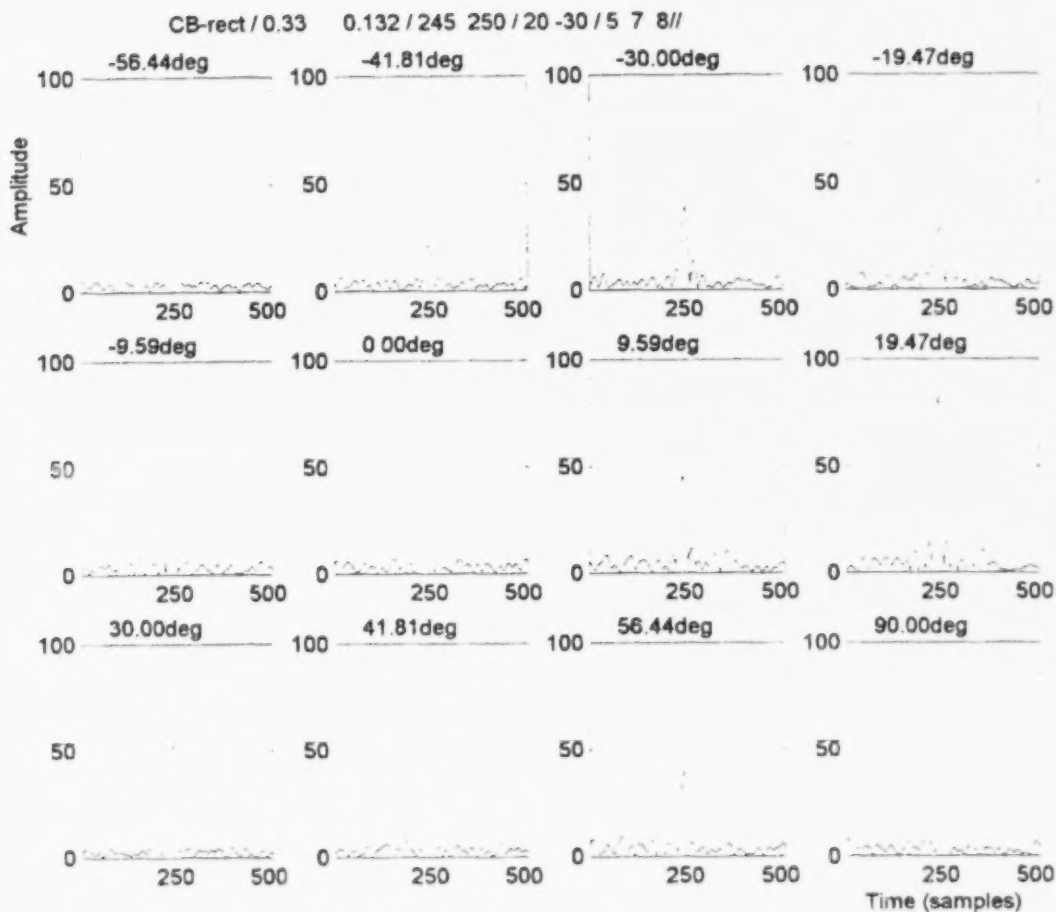


Fig 6.6(a) WB CB-RECT beamformer. Correlator output for two chirps with similar arrival times

Two chirp sources: Stronger at 20° , arrival time at sample 245; weaker at -30° , arrival time at sample 250. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples. $N=12$ sensors Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz Snapshot length = 128 time samples 12 beams Sensors [5 7 8] missing

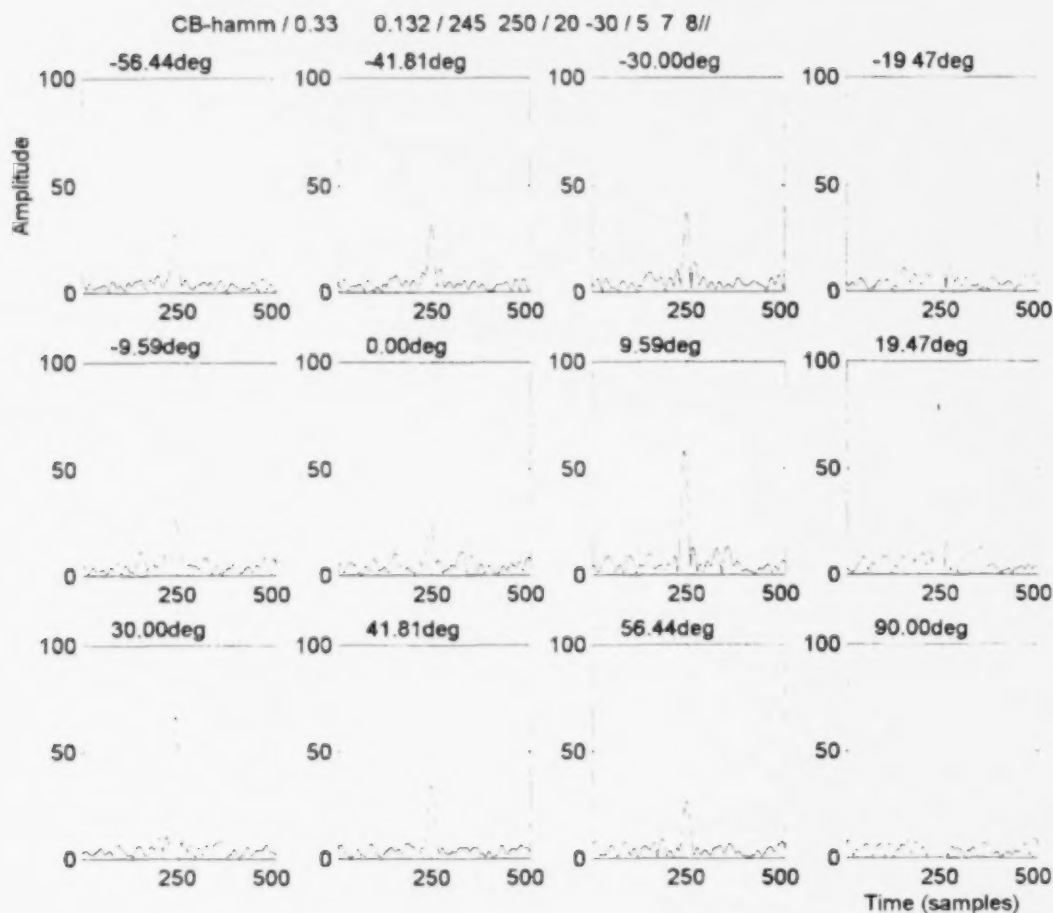


Fig. 6.6(b) WB CB-HAMM beamformer. Correlator output for two chirps with similar arrival times.

Two chirp sources: Stronger at 20° , arrival time at sample 245; weaker at -30° , arrival time at sample 250. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples $N=12$ sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. Sensors [5 7 8] missing.

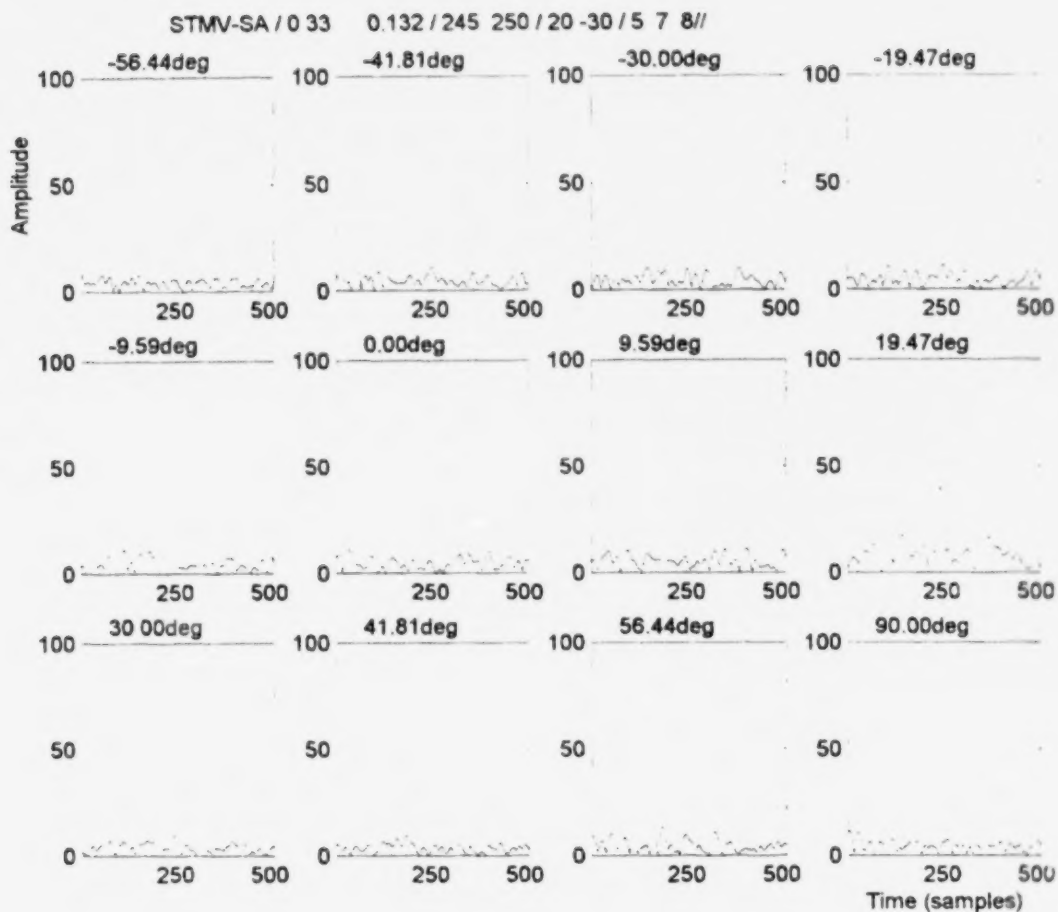


Fig. 6.6(c) WB STMV-SA beamformer. Correlator output for two chirps with similar arrival times. Note absence of weaker signal

Two chirp sources. Stronger at 20° , arrival time at sample 245; weaker at -30° , arrival time at sample 250. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples $N=12$ sensors. Spacing $d=1\text{m}$ Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. Sensors [5 7 8] missing.

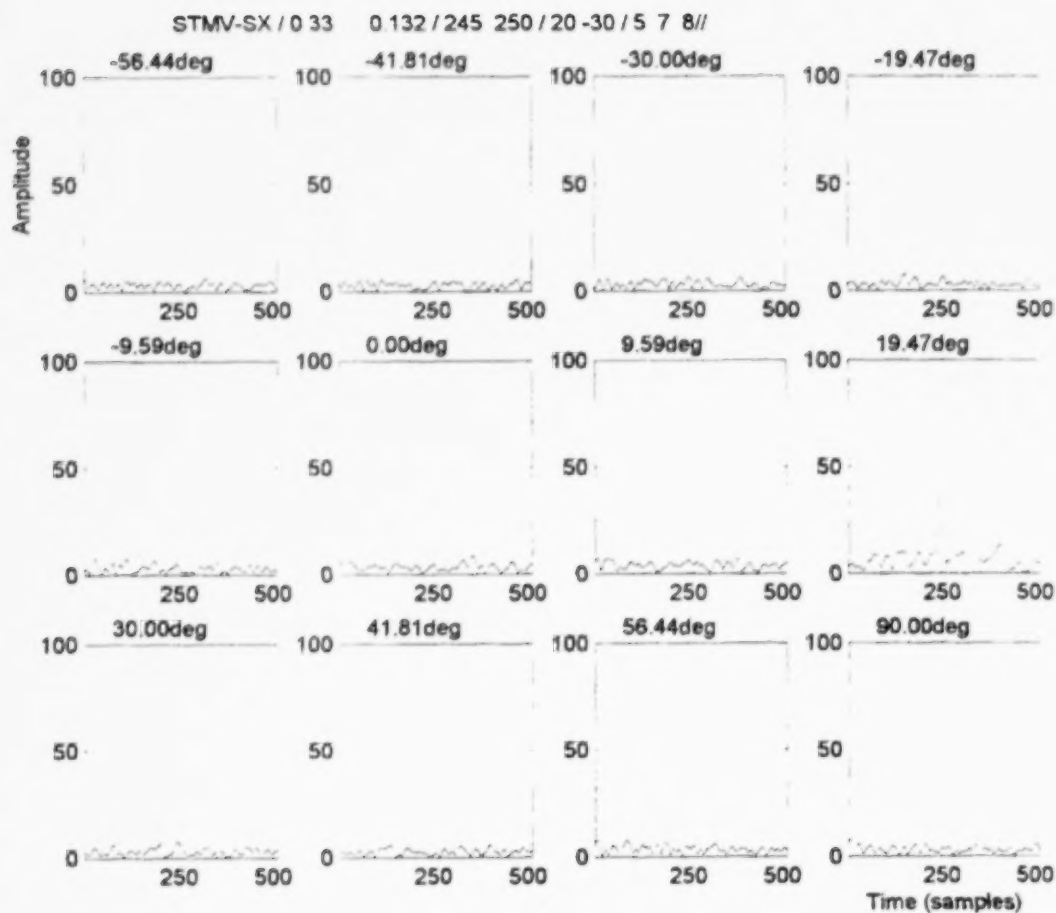


Fig. 6 6(d) WB STMV-SX beamformer. Correlator output for two chirps with similar arrival times. Note absence of weaker signal.

Two chirp sources: Stronger at 20° , arrival time at sample 245; weaker at -30° , arrival time at sample 250. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples. $N=12$ sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. Sensors [5 7 8] missing.

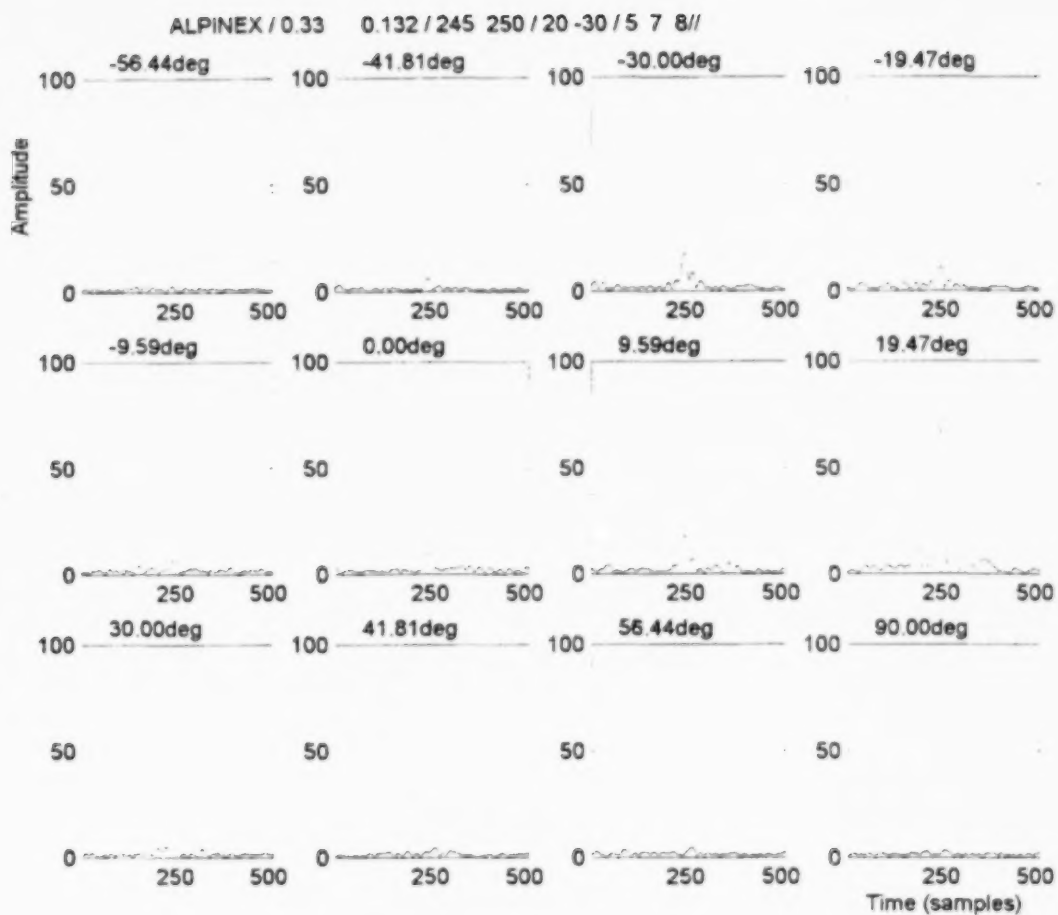


Fig 6.6(e) WB ALPINEX beamformer. Correlator output for two chirps with similar arrival times

Two chirp sources: Stronger at 20° , arrival time at sample 245, weaker at -30° , arrival time at sample 250. Linear chirp sweep from 313Hz to 375Hz over duration of 500 time samples. $N=12$ sensors. Spacing $d=1\text{m}$. Processing band 280-400Hz. Sampling rate = 1kHz. Snapshot length = 128 time samples. 12 beams. Sensors [5 7 8] missing.

7. CONCLUSION

This report has examined, in some depth, several aspects of the steered-minimum-variance, subarray beamforming method (STMV-SA) recently introduced at DREA by Stergiopoulos, working in collaboration with co-workers [2-2, 2-2, 2-3], for use with a uniform line-array. Initially, the question of subarray geometry was broached and while it was indeed found that the heavily overlapped, uniform subarray geometry of Stergiopoulos' team was preferable to the non-overlapped but contiguous subarray originally envisaged by Owsley [2-1], the very uniformity of the subarray structure could give rise to sidelobe-like artifacts, especially in the situations with missing sensors. However, this particular problem was found to be largely alleviated by using subarrays with randomised sensor locations.

Another avenue explored related to multipath scenarios. In the so-called NB mode with multiple tonal signals sharing the same frequency, there was the expected problem of signal cancellation which bedevils all MVDR-based adaptive beamformers in similar circumstances. However, it should be mentioned that useful NB beamforming was possible using the STMV-SA in non-multipath situations.

A negative result which was not originally anticipated, though perhaps obvious with hindsight, related to the matched-filter processing of WB chirps. It was discovered that chirp returns whose arrival times differed by less than the reciprocal of the frequency sweep multiplied by the ratio of the snapshot length to the chirp length, could also suffer a severe cancellation effect similar to the aforementioned NB multipath case. This implies that there is a potential multipath problem for the STMV-SA processor in both the NB and WB modes.

The issue of the coherence length of signals impinging on the array was also briefly investigated and it was found, within certain limits, to be of little practical significance.

An issue of much greater importance was that related to finite-data effects. This became a concern when the number of NB temporal data snapshots, or Fourier vectors, (equivalently, the time-bandwidth product of the processed data) approached the number of degrees of freedom in the adaptive processor, and this is precisely the case in the

"instantaneous convergence" application in this report, where only a single, relatively short, time-series observation is used to compute the adaptive array weights. The overall impact is that of limiting the output SNR available from the beamformer. During the course of this work it was deduced that previously published statistical results in this general area [4-1] could be slightly modified for use with the STMV-SA beamformer. One of the more interesting of them showed that, "all other things being equal", the STMV-SA beamformer in the NB mode could have an output SNR considerably higher than that of the corresponding classic NB MVDR beamformer with the same number of degrees of freedom and the same number of Fourier vectors. This is despite having only a single vector containing signal information involved in forming the underlying WB steered-covariance matrix.

One of the truly negative results stemming from this work concerned the Dominant-Mode Rejection Beamformer (DMRJ), which has recently been introduced as a computationally efficient approach to NB MVDR beamforming. It was found to be not at all well-suited for our STMV approach, which is based on the WB steered-covariance matrix. Of all the beamformers considered in this report, its performance was by far the worst. The problem was traced to the fact that, in the NB mode at least, only one Fourier vector contained signal information; there was none of the usual decorrelation via the implicit averaging of multiple signal bearing snapshots as is the case in classic NB MVDR beamforming. The performance of the DMRJ beamformer in our so-called NB mode was so poor, it was not even considered for WB processing. Considerable time and effort were spent ensuring that these problems were not the result of oversight or coding errors.

Finally, a software package was written to simulate the performance of several beamformers, adaptive and non-adaptive, in a variety of scenarios. These beamformers included the ALPINEX method introduced at DREA some years ago. The results were quite interesting. In the NB mode, and where there were no missing sensors, the non-adaptive Hamming-shaded beamformer seemed to be competitive, especially considering its simplicity. In situations with missing sensors, the ALPINEX method was clearly the overall NB method of choice.

In the WB mode, where there was matched-filter processing of FM chirp returns, the ALPINEX method was again the superior beamformer overall as STMV-SA processing was inferior in multipath scenarios. In the absence of multipath problems, the

STMV-SA method with randomised subarray geometry was perhaps marginally the best WB performer.

Overall, we can state that the STMV-SA approach with randomised subarrays is a relatively computationally efficient beamformer that is indeed reasonably successful in both the NB mode and the WB mode, provided neither involves multipath situations. However, it is not clear to this author whether its additional complexity and performance limitations would justify its use in preference to, say, a simple fixed-weight approach. The Hamming-shaded conventional beamformer, for instance, with its complete lack of multipath sensitivity, was usually very competitive in the experiments performed here. Additionally, these same experiments revealed that the STMV-SA performance is usually inferior to the somewhat less computationally efficient ALPINEX adaptive method which, interestingly, is also insensitive to multipath conditions.

It should be acknowledged that only a handful of simulations have been performed. It is not outside the bounds of possibility that other scenarios may show the STMV-SA method to be far superior to any other beamformer. However, based on the evidence to date, this seems to be very unlikely.

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The performance of the steered, minimum-variance, subarray beamformer is examined in some detail. It is demonstrated that improved performance is possible through the use of a randomised subarray geometry. It is also demonstrated that the beamformer is susceptible to multipath problems in both the narrowband and wideband modes. A series of simulation experiments allows its performance to be evaluated against that of other adaptive and non-adaptive beamformers.

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